DOCUMENT RESUME

ED 068 506 TM 001 844

AUTHOR	Andersen, Erling B.
TITLE	A Computer Program for Solving a Set of Conditional Maximum Likelihood Equations Arising in the Rasch Model for Questionnaires.
INSTITUTION	Educational Testing Service, Princeton, N.J.
REPORT NO	RM-72-6
PUB DATE	May 72
NOTE	53p.
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	*Computer Programs; Data Processing; Evaluation; Goodness of Fit; *Mathematical Models; Measurement Techniques; Models; *Programing; *Questionnaires; Research; *Statistical Analysis; Technical Reports; Test Interpretation
IDENTIFIERS	*Rasch Model

ABSTRACT

A computer program for solving the conditional likelihood equations arising in the Rasch model for questionnaires is described. The estimation method and the computational problems involved are described in a previous research report by Andersen, but a summary of those results are given in two sections of this paper. A working example is also provided to help the reader understand different computational steps of the program; these include (1) defining properties of the model, (2) computing estimates of the item parameters by taking the conditional distribution of the total set of responses given the individual score vectors as the likelihood function, (3) applying a goodness-of-fit test based on a grouping of possible score vectors, (4) developing technical, input-output, and main subroutines. Numerical examples are given of two actual executions of the program. The appendix, comprising nearly half of the report, lists the main program and subroutines. (LH)

ED 068506

RM-72-6

RESEARCH MEMORANDUM

A COMPUTER PROGRAM FOR SOLVING A SET OF CONDITIONAL MAXIMUM LIKELIHOOD EQUATIONS ARISING IN THE RASCH MODEL FOR QUESTIONNAIRES

Erling B. Andersen
Copenhagen School of Economics
and Business Administration

This Memorandum is for interoffice use. It is not to be cited as a published report without the specific permission of the author.

TM 001 844

Educational Testing Service
Princeton, New Jersey
May 1972

A Computer Program for Solving a Set of Conditional Maximum Likelihood Equations Arising in the Rasch Model for Questionnaires

Erling B. Andersen*

Copenhagen School of Economics

and Business Administration

1. Introduction

The Rasch model for questionnaires was suggested by Rasch (1960, 1961, 1966a, 1966b) in a series of papers. A main property of the model--as emphasized by Rasch--lies in the possibility of drawing conditional inferences about the parameters of the model. Some aspects of conditional inference related to the Rasch model were discussed by Andersen (1967, 1970a). In the latter paper the problem of estimating the question (or item) parameters by a conditional maximum likelihood method was considered. The main result is that conditional maximum likelihood estimators (c. m. l. estimators), defined as the item parameter estimates which maximize the conditional probability given minimal sufficient statistics for the individual parameters, are consistent and asymptotically normally distributed under the limiting process of increasing the number of individuals that answer the questionnaire. The Rasch model and applications thereof have also been discussed by Stene (1967); Anderson, Kearney, and Everett (1968); and Andersen (1970b).

In this paper we shall present a computer program for solving the conditional likelihood equations. The estimation method and the computational problems involved are described in a research report by Andersen

---

*Part of this research was carried out while the author visited Educational Testing Service in Fall 1970 and in August 1971.

(1969); for convenience, a summary of the results from that research report is given in sections 2 and 3 of this paper. This paper also contains a working example that may help the reader understand the different computational steps of the program.

2. Properties of the Model

The computer program has been written to handle the case in which there are m answer categories for each of k items. For the common testing situation where an answer to an item is considered to be either right or wrong, $m = 2$. We express a given individual's response to the i-th item when he chooses the j-th category with an m-dimensional vector having all elements equal to zero except the j-th element which is set equal to one. As a random variable, we indicate a response by $X_i = (X_{i1},\ldots,X_{im})$. The possible realizations of X_i are then $(1,0,\ldots,0)$, $(0,1,0,\ldots,0)$, $\ldots$, $(0,0,\ldots,1)$.

In the model we have two sets of parameters, individual parameters and item parameters. In the m-dimensional case we have for each individual a vector $(\tau_1,\ldots,\tau_m)$ of real valued individual parameters and for each item i a vector $(\theta_{i1},\ldots,\theta_{im})$ of real valued item parameters. The Rasch model (Rasch, 1961) provides that the event $\{X_{ij} = 1\}$ has probability

$$(1) \qquad P\{X_{ij} = 1\} = c_i^{-1} \exp(\theta_{ij} + \tau_j) \quad ,$$

where $c_i = \sum_{j=1}^{m} \exp(\theta_{ij} + \tau_j)$ since $P\{X_{i1} = 1\} + \ldots + P\{X_{im} = 1\} = 1$.

It is obvious from (1) that any constant can be subtracted from all θ's or from all τ's without changing the values of the probabilities. This indeterminacy in the parameterization can be removed, e.g., by putting $\tau_m = 0$ and $\theta_{im} = 0$ for all i. We note further that we can add any value α_j

to θ_{ij} if simultaneously α_j is subtracted from τ_j. This gives rise to the linear constraint $\sum_{i=1}^{k} \theta_{ij} = 0$. We observe finally that when we put $\theta_{im} = 0$ and $\tau_m = 0$, we can replace $(\tau_1,\ldots,\tau_m)$ with $(\tau_1,\ldots,\tau_m - 1)$ and $(\theta_{i1},\ldots,\theta_{im})$ with $(\theta_{i1},\ldots,\theta_{im} - 1)$ in the following.

The interpretation of the τ's and the θ's depends, of course, on the type of data we want to describe by the model. Two cases may, however, illustrate the situation. Suppose first that $m = 2$ and that the test is an achievement test. As we saw, we can put $\tau_2 = 0$ and $\theta_{i2} = 0$. The remaining parameters can be interpreted as τ_1 = the ability of the individual and θ_{i1} = the easiness of item i, or equivalently, $-\theta_{i1}$ = the difficulty of item i.

Suppose next that $m > 2$, that the test measures an attitude of the individuals and that the m answer categories range from a very positive response through $m - 2$ intermediate steps to a very negative response. In this case $\tau_1,\ldots,\tau_m$ can be interpreted as the given individual's inclination to answer with positive or negative responses. If, e.g., $\tau_1 > \tau_2 > \ldots > \tau_m$, the individual is likely to use positive answer categories more often than negative. In the same way $\tau_1 < \tau_2 < \ldots < \tau_m$ would indicate a very negatively reacting individual. Cases where, e.g., $\tau_1 < \tau_2 < \ldots < \tau_\nu, \tau_{\nu+1} > \tau_{\nu+2},\ldots,\tau_m$ for some ν between 2 and $m - 1$ would indicate an individual, who is likely to use the middle answer categories most frequently. We can apply the same sort of considerations to $\theta_{i1},\ldots,\theta_{im}$. If, e.g., $\theta_{i1} > \theta_{i2} > \ldots > \theta_{im}$, we should expect item

i to provoke positive answers from the individuals more often than it provokes negative answers.

Let $x_i = (x_{i1},\ldots,x_{im})$ be the observed values of $(X_{i1},\ldots,X_{im})$. Then it is easy to see that (1) is equivalent to

$$(2) \qquad P\{(X_{i1},\ldots,X_{im}) = (x_{i1},\ldots,x_{im})\} = c_i^{-1} \exp\Big(\sum_{j=1}^{m-1} (\theta_{ij} + \tau_j)x_{ij}\Big) \quad ,$$

or letting $f(x_{i1},\ldots,x_{im}) = P\{(X_{i1},\ldots,X_{im}) = (x_{i1},\ldots,x_{im})\}$

$$(3) \qquad f(x_{i1},\ldots,x_{im}) = c_i^{-1} \exp\Big(\sum_{j=1}^{m-1} (\theta_{ij} + \tau_j)x_{ij}\Big) \quad .$$

If, for a given individual, the responses on the different items are stochastically independent, the probability of an individual's set of k responses is specified by

$$(4) \qquad f(x_{11},\ldots,x_{km}) = \prod_{i=1}^{k} f(x_{i1},\ldots,x_{im})$$

$$= \Big(\exp\Big(\sum_{j=1}^{m-1} \tau_j x_{\cdot j}\Big)\Big)\Big(\prod_{i=1}^{k} c_i^{-1}\Big) \exp\Big(\sum_{i=1}^{k}\sum_{j=1}^{m-1} \theta_{ij}x_{ij}\Big) \quad ,$$

where $x_{\cdot j} = \sum_{i=1}^{k} x_{ij}$. Since for a given i, $x_{ij} = 1$ if answer category j is used on item i and 0 otherwise, it is easily seen that $x_{\cdot j}$ is the number of times answer category j has been used on the k items. In an attempt to conform with the usual psychometric terminology let us call $(x_{\cdot 1},\ldots,x_{\cdot m})$ the score vector of an individual. Since $f(x_{11},\ldots,x_{km})$ is a multiparameter exponential family it follows from Neyman's factorization lemma that $(X_{\cdot 1},\ldots,X_{\cdot m})$, with $X_{\cdot j} = \sum_{i=1}^{k} X_{ij}$, $j = 1,\ldots,m$, are a set

of minimal sufficient statistics, for the individual parameters $(\tau_1,\ldots,\tau_m)$. This means, according to the definition of sufficient statistics, that the distribution of an individual's total set of responses $(X_{11},\ldots,X_{km})$ given his score vector $(X_{\cdot 1},\ldots,X_{\cdot m})$ depends only on the item parameters and not on his individual parameters.

As we shall see in section 3 the appropriate way to estimate the item parameters is a conditional maximum likelihood method, i.e., to take the conditional probability of the responses given the values of the minimal sufficient statistics for the individual parameters as the likelihood function. In the remaining part of this section we shall derive these conditional probabilities.

Let $r = (r_1,\ldots,r_m)$ be one of the possible score vectors $(x_{\cdot 1},\ldots, x_{\cdot m})$. We note that r is a vector with nonnegative components satisfying $\sum_{j=1}^{m} r_j = k$. Let Θ be the matrix $[\theta_{ij}]$, $i = 1,\ldots,k$; $j = 1,\ldots, m - 1$. Then we define

$$(5) \qquad \gamma(r_1,\ldots,r_m;\Theta) = \sum_{x_{\cdot j}=r_j;j=1,\ldots,m} \exp\Big(\sum_{i=1}^{k}\sum_{j=1}^{m-1} \theta_{ij}x_{ij}\Big) ,$$

i.e., $\gamma(r;\Theta)$ is the summation of $\exp\big(\sum_{i=1}^{k}\sum_{j=1}^{m-1} \theta_{ij}x_{ij}\big)$ over all sets of responses $(x_{11},\ldots,x_{km})$ for which the score vector $(x_{\cdot 1},\ldots,x_{\cdot m}) = (r_1,\ldots,r_m)$. If we picture $(x_{11},\ldots,x_{km})$ as a matrix $[x_{ij}]$, $i = 1,\ldots,k$; $j = 1,\ldots,m$ the summation in (5) is extended over all matrices that have the same column sums $x_{\cdot 1} = r_1,\ldots,x_{\cdot m} = r_m$. Let

$\phi(x_{11},\ldots,x_{km}|x_{\cdot 1},\ldots,x_{\cdot m})$ be the conditional probability $P\{(X_{11},\ldots,X_{km}) = (x_{11},\ldots,x_{km})|(X_{\cdot 1},\ldots,X_{\cdot m}) = (x_{\cdot 1},\ldots,x_{\cdot m})\}$. We then have

$$(6) \qquad \phi(x_{11},\ldots,x_{km}|x_{\cdot 1},\ldots,x_{\cdot m}) = \exp\Big(\sum_{i=1}^{k}\sum_{j=1}^{m-1}\theta_{ij}x_{ij}\Big)/\gamma(x_{\cdot 1},\ldots,x_{\cdot m};\Theta) \quad .$$

This expression can easily be derived from the fact that (4) is multiparameter exponential family. In fact (6) is a special case of a formula given by Lehmann (1959, lemma 6, p. 47).

So far we have only considered the responses of one individual. Suppose that N individuals have answered the k items and let the individuals be indexed $p = 1,\ldots,N$. Let $(x_{11}^{(p)},\ldots,x_{km}^{(p)})$ be the responses of individual p , and $(x_{\cdot 1}^{(p)},\ldots,x_{\cdot m}^{(p)})$ be the score vector of individual p . Then assuming independence between responses from different individuals, from (6) we obtain the simultaneous conditional distribution of the N individuals' responses given their score vectors as

$$(7) \qquad \prod_{p=1}^{N}\phi(x_{11}^{(p)},\ldots,x_{km}^{(p)}|x_{\cdot 1}^{(p)},\ldots,x_{\cdot m}^{(p)})$$

$$= \exp\Big(\sum_{p=1}^{N}\sum_{i=1}^{k}\sum_{j=1}^{m-1}\theta_{ij}x_{ij}^{(p)}\Big)/\prod_{p=1}^{N}\gamma(x_{\cdot 1}^{(p)},\ldots,x_{\cdot m}^{(p)};\Theta) \quad .$$

This expression can be simplified in two ways. First, θ_{ij} does not depend on p and consequently the triple sum reduces to $\sum_{i=1}^{k}\sum_{j=1}^{m-1}\theta_{ij}y_{ij}$, where $y_{ij} = \sum_{p=1}^{N}x_{ij}^{(p)}$. Secondly there are usually several individuals who have the same score vector. Instead of extending the product in the

denominator of (7) over all individuals we can alternatively extend the product over all possible values of score vectors. If we denote the number of individuals with score vector $r = (r_1,\ldots,r_m)$ by n_r the product becomes

$$\prod_{r=(0,0,\ldots,k)}^{(k,0,\ldots,0)} \gamma^{n_r}(r_1,\ldots,r_m;\theta) \quad ,$$

where $r = (k,0,\ldots,0)$ up to $(0,0,\ldots,k)$ means that the product is extended over all possible score vectors $(r_1,\ldots,r_m)$. At this stage it does not matter in what order we take the r's in the product. In the computer program a special ordering of the possible values of the r's has been adopted. This specific ordering is described in section 5. Collecting the results we have

$$(8) \qquad \prod_{p=1}^{N} \phi(x_{11}^{(p)},\ldots,x_{km}^{(p)} | x_{\cdot 1}^{(p)},\ldots,x_{\cdot m}^{(p)})$$

$$= \exp\Big(\sum_{i=1}^{k}\sum_{j=1}^{m-1} \theta_{ij}y_{ij}\Big) \Big/ \prod_{r=(0,0,\ldots,k)}^{(k,0,\ldots,0)} \gamma^{n_r}(r_1,\ldots,r_m;\theta) \quad .$$

Equation (8) is the basic probability to be used for conditional maximum likelihood estimation of the θ's in the next section.

3. The Likelihood Equations

The main purpose of the present program is to compute estimates of the item parameters. We shall obtain the estimates as conditional maximum likelihood estimates, i.e., we shall take the conditional distribution (8) of the total set of responses given the individual score vectors as our likelihood function. This conditional distribution has the advantage of not being dependent upon the values of the individual parameters. We could have taken the unconditional distribution of the responses as our likelihood function but we would have had the additional problem of estimating the individual parameters. As shown in Andersen (1970b) a simultaneous maximum likelihood estimation of both item and individual parameters would have led to an inconsistent estimation of the item parameters. It follows from results in Andersen (1970a) that a conditional maximum likelihood estimation of the item parameters leads to consistent estimates. We therefore take the set of θ -values that maximize (8) as our estimates for the θ_{ij} . Differentiation of the logarithm of (8) gives the (conditional) likelihood equations

$$(9) \quad y_{ij} = \sum_{r=(0,0,\ldots,k)}^{(k,0,\ldots,0)} n_r \, \partial \log \gamma(r_1,\ldots,r_m;\theta)/\partial\theta_{ij} \quad ,$$

$$i = 1,\ldots,k \; ; \quad j = 1,\ldots,m-1 \quad .$$

In order to solve (9) we need a handy expression for $\partial \log \gamma(r_1,\ldots,r_m;\theta)/\partial\theta_{ij}$. From the definition (5) of $\gamma(r_1,\ldots,r_m;\theta)$ we get

$$\partial\gamma(r_1,\ldots,r_m;\Theta)/\partial\theta_{ij}$$

$$= \sum_{x_{\cdot q}=r_q,\, q=1,\ldots,m} x_{ij} \exp\Big(\sum_{s=1}^{k}\sum_{q=1}^{m-1} x_{sq}\theta_{sq}\Big) \quad ,$$

where i and j are indices which indicate the specific θ_{ij} with respect to which we are differentiating; s and q are the indices used to indicate summations over the range of the dimensions of x_{sq} and θ_{sq}.

The summation on the right-hand side of the equation above is over all matrices x_{sq}, $s = 1,\ldots,k$, $q = 1,\ldots,m$ for which $x_{\cdot q} = r_q$, $q = 1,\ldots,m$. We can separate this summation into two parts, one that consists of all x_{sq}-matrices which have 1 in the i,j-th position, and one that consists of all x_{sq}-matrices with a 0 in the i,j-th position. Because of the factor x_{ij}, the latter part becomes 0. Since x_{ij} must be 1 in the first part, it consists of all x-matrices, for which $\sum_{s\neq i}^{k} x_{sq} = r_q$ for $q \neq j$ and $\sum_{s\neq i}^{k} x_{sj} = r_j - 1$.

Let $B^{(ij)}$ denote the set of x-matrices that satisfy $\sum_{s\neq i}^{k} x_{sq} = r_q$ for $q \neq j$ and $\sum_{s\neq i}^{k} x_{sj} = r_j - 1$. We then have

$$\partial\gamma(r_1,\ldots,r_m;\Theta)/\partial\theta_{ij} = \sum_{B^{(ij)}} \exp\Big(\theta_{ij} + \sum_{s\neq i}^{k}\sum_{q=1}^{m-1} x_{sq}\theta_{sq}\Big)$$

$$= \exp(\theta_{ij})\gamma(r_1,\ldots,r_j - 1,\ldots,r_m;\Theta^{(i)}) \quad ,$$

where $\Theta^{(i)}$ is Θ with the i-th row deleted. Consequently we have .

$$(10) \qquad \partial \log \gamma(r_1,\ldots,r_m;\Theta)/\partial\theta_{ij} = \exp(\theta_{ij}) \cdot \gamma(r_1,\ldots,r_j - 1,\ldots,r_m;\Theta^{(i)})/\gamma(r_1,\ldots,r_m;\Theta) \quad .$$

In order to solve (9) we need some convenient way of computing the γ -ratios on the right-hand side of (10). In the program the Newton-Raphson iteration method is used to solve (9). To employ this method, we need second derivatives of $\log \gamma(r_1,\ldots,r_m;\Theta)$ with respect to all the θ's. These second derivatives can be written in terms of the same type of γ -ratios as those shown in (10). Further details on how to compute the γ -ratios and also details on the solution of (9) are given in Andersen (1969).

From (9) it follows that the data we need in order to obtain estimates of the item parameters are the item totals y_{ij} ; $i = 1,\ldots,k$, $j = 1,\ldots,m$ and for each score vector $r = (r_1,\ldots,r_m)$ the number of individuals who have obtained this score vector. In the program the y_{ij}'s are stored in matrix IA and the values of n_r are stored in vector N . A natural ordering of the possible score vectors is described in section 4. This ordering is used to determine where in N a specific n_r is stored.

4. A Goodness of Fit Test for the Model

The goodness of fit test we shall apply is based on a grouping of possible score vectors. Let R_0 be the set of all possible score vectors an individual can obtain and let $R_1,\ldots,R_n$ be n disjoint subsets of R_0, such that

$$(11) \qquad \bigcup_{q=1}^{n} R_q = R_0 \quad .$$

Consider one of these subsets R_q. Obviously R_q determines a subset P_q of the population by the rule that an individual is in P_q if his score vector satisfies $(x_{\cdot 1},\ldots,x_{\cdot m}) \in R_q$. We can now compute restricted estimates of the item parameters by considering only individuals in P_q. If namely $y_{ij}^{(q)}$ denotes the number of times the individuals in P_q have used answer category j on item i, we, corresponding to (9), have the restricted likelihood equations

$$(12) \qquad y_{ij}^{(q)} = \sum_{(r_1,\ldots,r_m)\in R_q} n_r \, \partial \log \gamma(r_1,\ldots,r_m;\theta)/\partial\theta_{ij} \quad .$$

Since (12) does not involve the individual parameters we should expect to get just as good estimates of the θ's from (12) as from (9); of course, the precision of the estimates is somewhat lower. We shall, therefore, regard the model as a good fit to the data if the solutions $\hat{\theta}_{ij}^{(q)}$; $i = 1,\ldots,k$, $j = 1,\ldots,m-1$ to (12) for different values of q show only nonsystematic variations from the solutions $\hat{\theta}_{ij}$; $i = 1,\ldots,k$, $j = 1,\ldots,m-1$ to (9). In order to determine whether the derivations we

find can be regarded as due to random errors alone, we consider the following test statistic.

$$(13)\qquad Z = -2 \sum_{i=1}^{k}\sum_{j=1}^{m} y_{ij}\hat{\theta}_{ij} + 2 \sum_{(r_1,\ldots,r_m)\in R_0} n_r \log \gamma(r_1,\ldots,r_m;\hat{\Theta}) + \sum_{q=1}^{k} 2 \sum_{i=1}^{k}\sum_{j=1}^{m} y_{ij}^{(q)}\hat{\theta}_{ij}^{(q)} - 2 \sum_{q=1}^{k} \sum_{(r_1,\ldots,r_m)\in R_q} n_r \log \gamma(r_1,\ldots,r_m;\hat{\Theta}^{(q)}) \quad ,$$

where $\hat{\Theta} = [\hat{\theta}_{ij}]$ and $\hat{\Theta}^{(q)} = [\hat{\theta}_{ij}^{(q)}]$. Since

$$(14)\qquad L(\Theta) = \sum_{i=1}^{k}\sum_{j=1}^{m} y_{ij}\theta_{ij} - \sum_{r_1,\ldots,r_m} n_r \log \gamma(r_1,\ldots,r_m;\Theta)$$

is the logarithm of the conditional likelihood function (8), and

$$L_q(\Theta) = \sum_{i=1}^{k}\sum_{j=1}^{m} y_{ij}^{(q)}\theta_{ij} - \sum_{(r_1,\ldots,r_m)\in R_q} n_r \log \gamma(r_1,\ldots,r_m;\Theta)$$

is the logarithm of the restricted likelihood function, Z can be expressed by

$$Z = -2 \log\{L(\hat{\Theta}) / \prod_{q=1}^{n} L_q(\hat{\Theta}^{(q)})\} \quad .$$

Consequently Z is a sort of transformed likelihood ratio test statistic, where we maximize the likelihood under the model by the overall estimates $\hat{\theta}_{ij}$ and maximize the likelihood without the model by restricted estimates

for each of the subgroups of individuals. It can be shown (Andersen (1971)) that Z, if the model is true, can be approximated by a χ^2 distributed random variable with $f = (n - 1)(k - 1)(m - 1)$ degrees of freedom. We reject the model for large values observed of Z.

5. Technical Subroutines

A number of subroutines have been developed to take care of the technicalities. We first consider the summation over r in (9). Let R_0 be the set of all m-dimensional vectors $(r_1,\ldots,r_m)$ with integer components satisfying $\sum_{p=1}^{m} r_p = k$. We need a complete ordering of R_0 in order to perform a summation over r. The ordering of R_0 adopted here has $(0,0,\ldots,k)$ as first element and $(k,0,\ldots,0)$ as last element and is otherwise given by the rule that a step from $(r_1,\ldots,r_m)$ to $(r_1,\ldots,r_p+1,\ldots,r_q-1,\ldots,r_m)$ where $p<q$ brings us to a higher order r. In the program, r's are generally denoted by R, occasionally by IR; m is denoted by M and k by K. Subroutine VEC has R, KEY and M as arguments, where KEY is an error index. When VEC is called with a given vector R, it returns with the next R in the complete ordering of R_0. Thus consecutive calls of VEC will execute a run through all the possible values of r. We shall in the sequel refer to this process as an r-run. Any summation over the elements of R_0 is thus executed in the program as an r-run.

To avoid rounding errors that for large k typically occur halfway through an r-run, the present version of the program reverses the call of VEC when the last r for which $r_m = k/2$ is reached. Thus the program starts with $r = (0,0,\ldots,k)$ and works forward using VEC until $r = (k - k/2, 0,\ldots,0,k/2)$. Then restarting with $r = (k,0,\ldots,0)$ the program works backwards through the r-listing by calling AVEC. Thus AVEC with the same arguments as VEC will change r to r' if VEC changes r' to r.

The elements of R_0 are indexed with the integers from 1 to $\binom{m+k-1}{m-1}$ in accordance with the ordering described above. If we denote by $R_0(c)$

the subset of R_0 for which $r_m = c, R_0(c)$ has been indexed according to the induced ordering. Thus $(0,0,\ldots,k)$ and $(k,0,\ldots,0)$ have indices 1 and $\binom{m+k-1}{m-1}$ in R_0 and $(0,0,\ldots,k-c)$ and $(k-c,\ldots,0)$ have indices 1 and $\binom{k-c+m-2}{m-2}$ in $R_0(c)$. To identify the index of any r in R_0 and $R_0(c)$, subroutine INDEX with arguments R, IC and M as arguments has been constructed. If IC = 0, INDEX is equal to the index of R in R_0. If IC > 0, INDEX is equal to the index of R in $R_0(c)$ with C = IC.

Binomial coefficients are used frequently throughout the program. BINOM is a function subroutine with integer arguments IX and IN, such that $\binom{n}{x}$ = BINOM(IX,IN) for x = IX and n = IN.

One of the possibilities in the program is to generate data from random numbers. Subroutines GAUSS and RANDU have been taken from the I.B.M. Scientific Subroutine Packages to which the reader is referred for details. GAUSS generates random deviates from a standard normal distribution and RANDU generates random numbers between 0 and 1.

Subroutine PROB generates random numbers from a given multinormal distribution: If P(1),...,P(M) are the M probabilities of the distribution, a call of PROB(IX,P,M,J) will return IX = j if a random number X between 0 and 1 generated from RANDU satisfies $P(j-1) < X \leq P(j)$ with P(0) = 0.

In order to solve the likelihood equations we need a routine for computing the inverse of a matrix. In the present version of the program CHOLES is such a subroutine.

6. Input-Output Subroutines

The program has four input subroutines READ1, READ2, READ3 and DATASU. In each of these the basic data matrices IA and N are generated. If I = i is a particular item and J = j is an answer category, then IA(I,J) is the total number of times the individuals have used answer category j on item i . If L is the index of a score vector $r = (r_1,\ldots,r_m)$ according to the complete ordering of R_0 defined in section 5, then N(L) is the number of individuals that have obtained score vector r .

READ1 carries out the reading of individual data. Prior to the reading of the data cards, three cards must appear containing tape number to read from, the reading format and a 4 x 3 integer matrix IB that specifies the coding of answer categories. Then follows one record for each individual. By one record is meant one record on the input tape or one card if the input tape is no. 5. For each record the program reads a k -dimensional integer vector IAA . IAA is then translated into IVEC by the rule: If the value of IAA(J) is equal to IB(L,LL) for any L and LL then IVEC(J) is put equal to L . This allows us to let as many as three different characters on the input tape be coded by the same integer. The program reads data for NA individuals and writes a record for each individual on tape 98. Each record contains a serial number (NA for the first, 1 for the last read person), the content of IVEC and a score value IN defined as IN = INDEX(IR) , where IR is the individual's score vector.

READ2 takes data in compact form, i.e., the program reads N and IA directly. The reading format is (20I4).

READ3 is specially designed for model control. Prior to the call of READ3, an integer vector NKO of the same length as N is read. If in the J^{th} call of READ3, NKO(I) $\neq$ J the reading procedure skips all persons for which the index of his score vector is I . The reading is from tape 98. READ1 must have been called, therefore, prior to READ3. READ3 permits us to estimate the parameters for any subset of the individuals corresponding to a subset of R_0 . In particular we can compare the estimates from individuals with high and low scores.

Finally, DATASU contains the reading procedure for randomly generated data. In the present version, we considered item parameters with structure $\theta_{ij} = \theta_i \phi_j$ and individual parameters with structure $\tau_{pj} = \tau_p \phi_j + \rho_j$. , where the values of $\theta_1,\ldots,\theta_k$, $\phi_1,\ldots,\phi_m$ and $\rho_1,\ldots,\rho_m$ are read by the program prior to the data generating. The individual parameters $\tau_1,\ldots,\tau_N$ are defined as a sample of N random deviates from a normal distribution with mean 0 and standard deviation 1.

The program contains three main writing parts.

WRITE1 has as output the matrices N and IA after each reading routine has been completed.

WRITE2 gives for each iteration of the estimation procedure the obtained parameter values and the improvements from the previous iteration.

Finally, implicit in PSIMQ, which we shall describe later, is printing of the inverse to the matrix CC ,· the second derivatives of the likelihood function. This matrix is an approximation to the covariance matrix of the estimates, and the square roots of the diagonal elements are approximations to the standard errors of the estimates. These numbers are also printed in PSIMQ.

7. The Main Subroutines

The program has two control procedures. Subroutine MAIN contains the input of

NA = the number of individuals

M = the number of answer categories

K = the number of items

JTEST = an indicator of the type of reading we want.

From MAIN the general supervising procedure SUB is called. All further steps in the program are executed by calls from SUB.

The choice of input form is governed by the indicator variable JTEST as follows:

For JTEST = -2 the program generates random data and in addition groups the data, i.e., calls DATASU and READ3 in turn.

For JTEST = -1 the program generates random data, i.e., calls DATASU.

For JTEST = 0 the program reads on compact form, i.e, calls READ2.

For JTEST = +1 the program reads on individual form, i.e., calls READ1.

For JTEST = +2 the program reads grouped data, i.e., calls READ1 and READ3 in turn.

After the input of IA and N has been completed INTIAL is called. In this routine initial values for the estimators are computed.

Next IT is put equal to 0 and the iterative estimation procedure starts. In each iteration ESTIM and PSIMQ are called.

In ESTIM the values of the differences between the right- and left-hand sides of (9) with θ_{ij} replaced by $\hat{\theta}_{ij}$ are computed and stored in the $(k - 1)(m - 1)$ -dimensional vector F . The element corresponding to (i,j)

is stored as the $(i - 1)(m - 1) + j^{th}$ element of F. The derivatives of the right-hand side of (9) are also computed in ESTIM and stored in the $(k - 1)(m - 1)$ -dimensional square matrix CC. The derivative with respect to θ_{pq} is stored in the $(i - 1)(m - 1) + j^{th}$ row and $(p - 1)(m - 1) + q^{th}$ column of CC. The method for computing F and CC is described in detail in Andersen (1969). Since ESTIM is rather large, a few comments have been added to the listing to explain where in the routine the different steps are executed.

The calling arguments of ESTIM are IA, N and $DELTA$, where the km -dimensional matrix $DELTA$ contains the values of $\exp(\theta_{ij})$ $i = 1,\ldots,k$, $j = 1,\ldots,m$,

The improvements of the θ's are now given as the solutions to the matrix equation

$$-CC \cdot X = F \quad , \tag{15}$$

where CC is a $(k - 1)(m - 1)$ -dimensional square matrix and X and F are $(k - 1)(m - 1)$ -dimensional vectors. If $\theta_{ij}^{(t)}$, $i = 1,\ldots,k - 1$, $j = 1,\ldots,m - 1$ are the estimates after t iterations, and CC and F are computed with these values of the θ's as arguments we solve (15) and get improved estimates $\theta_{ij}^{(t+1)}$ as

$$\theta_{ij}^{(t+1)} = \theta_{ij}^{(t)} + x_{ij}$$

where x_{ij} is the $(i - 1)(m - 1) + j^{th}$ element of X.

The solution of (15) takes place in PSIMQ. PSIMQ has CC and F as arguments and returns X in F.

After the call of PSIMQ the absolute values of the elements of F are compared with TOL = 0.005 . If all the elements of F are less than TOL in absolute value the iterative procedure is stopped.

The necessary computations for the goodness of fit test described in section 4 takes place in subroutine LOGLIK. LOGLIK has the matrices EPS , G , IA and ALR as arguments. IA is defined above. EPS is a $k \cdot m$ -dimensional matrix with elements θ_{ij} ; $i = 1,\ldots,k$, $j = 1,\ldots,m$. G contains the quantities

$$(16) \qquad g(r_1,\ldots,r_m;j) = \gamma(r_1,\ldots,r_j - 1,\ldots,r_m;\Theta)/\gamma(r_1,\ldots,r_m;\Theta) \quad .$$

When $(IR(1),\ldots,IR(M)) = (r_1,\ldots,r_m)$ and $i = INDEX(IR)$, $g(r_1,\ldots, r_m;j)$ is stored as the $(i,j)^{th}$ element of G .

The quantities (16) are computed as a by-product of the estimation procedure in ESTIM. On return from LOGLIK the variable ALR contains the value of the right-hand side of (14).

8. Numerical Examples

We shall now in more detail study two actual executions of the program. Our first example has parameters $k = 8$ and $m = 4$ and is based on randomly generated data. We choose the following set of parameters.

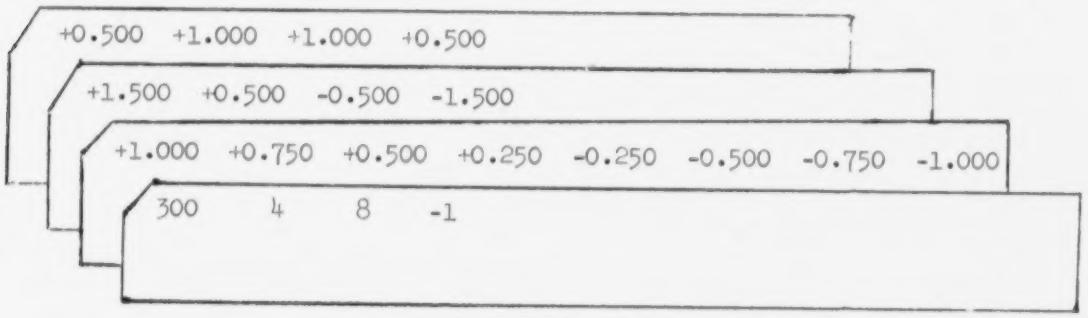
$$(\theta_1,\ldots,\theta_8) = (1.0,\ 0.75,\ 0.5,\ 0.25,\ -0.25,\ -0.5,\ -0.75,\ -1.0)$$

$$(\phi_1,\ldots,\phi_4) = (1.5,\ 0.5,\ -0.5,\ -1.5)$$

and

$$(\rho_1,\ldots,\rho_m) = (0.5,\ 1.0,\ 1.0,\ 0.5)\ .$$

The control cards corresponding to these parameters and 300 random individuals are



JTEST = -1 on the first card tells the program to call DATASU, where the generation of data takes place. Table 1 shows the generated data, i.e., the question-totals $IA(J,L)$, $J = 1,\ldots,k$, $L = 1,\ldots,m$, where $n_1 = \binom{k + m - 1}{m - 1}$ and the person-totals $N(I)$, $I = 1,\ldots,n_1$. The data are shown as displayed by the program, except that we show only the first person-groupings to save space. Columns 2 to 5 under heading PERSON TOTALS are the values of $r_1,\ldots,r_4$. The last column is n_r.

Table 1. Generated Data for 300 Individuals

QUESTION TOTALS

QUESTIONS	CATEGORIES			
	1	2	3	4
1	149	68	47	36
2	142	68	51	39
3	114	58	67	61
4	96	72	59	73
5	73	58	72	97
6	49	47	76	128
7	42	43	93	122
8	28	46	82	144

PERSON TOTALS

SCORE VECTOR	r_1	r_2	r_3	r_4	n_r
1	0	0	0	8	3
2	0	0	1	7	3
3	0	1	0	7	4
4	1	0	0	7	2
5	0	0	2	6	9
6	0	1	1	6	5
7	1	0	1	6	3
8	0	2	0	6	2
9	1	1	0	6	0
10	2	0	0	6	1
11	0	0	3	5	3
12	0	1	2	5	6
13	1	0	2	5	2
14	0	2	1	5	4
15	1	1	1	5	2

In Table 2 the output after 0 iterations, i.e., the initial parameter estimates, and the output after the final iterations, i.e., the obtained conditional maximum likelihood estimates, are shown as displayed by the program. The writing is from WRITE2. The matrix DELTA contains the values of $\delta_{ij} = \exp(\theta_{ij})$. Matrix EPSILON contains the values of θ_{ij}.

Table 2. Initial and Final Parameter Estimates for the Data in Table 1

ITERATION 0
IMPROVEMENTS

0.0000	0.0000	0.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000

DELTA

4.3167	2.6053	1.5241
3.7975	2.4049	1.5266
1.9491	1.3114	1.2822
1.3716	1.3604	0.9435
0.7849	0.8247	0.8665
0.3993	0.5065	0.6931
0.3591	0.4861	0.8899
0.2028	0.4406	0.6648

EPSILON

1.4625	0.9575	0.4214
1.3343	0.8775	0.4230
0.6674	0.2711	0.2486
0.3160	0.3078	-0.0582
-0.2422	-0.1927	-0.1433
-0.9181	-0.6803	-0.3665
-1.0243	-0.7213	-0.1167
-1.5955	-0.8196	-0.4083

ITERATION 4
IMPROVEMENTS

0.0000	0.0000	0.0000
0.0000	0.0000	0.0000
0.0000	0.0000	-0.0000
0.0000	0.0000	-0.0000
0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000

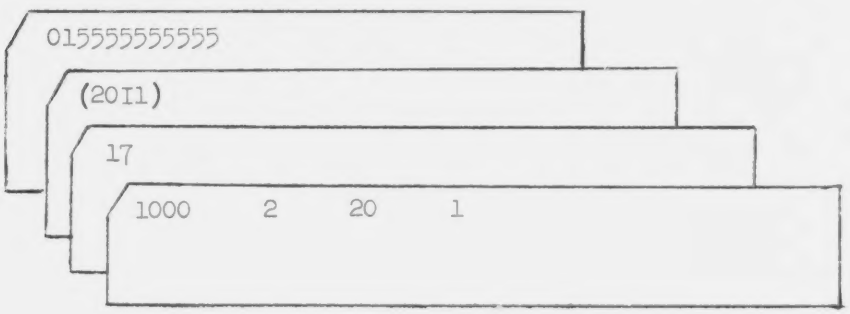
DELTA

13.8238	5.2496	2.2584
11.0449	4.5805	2.1895
3.3915	1.8073	1.5182
1.9207	1.6386	1.0381
0.6627	0.7309	0.8031
0.1822	0.3137	0.5305
0.1471	0.2893	0.6699
0.0566	0.2117	0.4496

EPSILON

2.6268	1.6581	0.8146
2.4020	1.5218	0.7037
1.2213	0.5919	0.4175
0.6527	0.4938	0.0374
-0.4114	-0.3134	-0.2193
-1.7026	-1.1592	-0.6339
-1.9164	-1.2402	-0.4006
-2.8723	-1.5528	-0.7995

As our second example we shall consider data from the Scholastic Aptitude Test, verbal part, which have earlier been treated by Lord (1968) based on Birnbaum's three parameter logistic model. The original test had 80 test items each with two answer categories: a correct and an incorrect answer. For convenience we consider only the first twenty items from the test battery. We shall assume that the input tape is mounted on tape unit 17 and that we want to analyze the responses of the first 1000 individuals on the tape. The following set of control cards will then initiate a conditional maximum likelihood estimation of the item parameters.



On the fourth card only the first two integers are important. They indicate for the program that 0 is coded as 1 and 1 is coded as 2. Table 3 shows the resulting data. The writing takes place in WRITE1.

Table 3. Data from SAT Test, Verbal Part

QUESTION	TOTALS	
1	140	860
2	223	777
3	239	761
4	315	685
5	541	459
6	537	463
7	390	610
8	419	581
9	668	332
10	691	309
11	77	923
12	206	794
13	268	732
14	425	575
15	696	302
16	713	287
17	685	315
18	720	280
19	784	216
20	726	274

PERSON	TOTALS		
1	0	20	18
2	1	19	25
3	2	18	25
4	3	17	41
5	4	16	46
6	5	15	57
7	6	14	56
8	7	13	53
9	8	12	55
10	9	11	69
11	10	10	95
12	11	9	93
13	12	8	95
14	13	7	81
15	14	6	82
16	15	5	58
17	16	4	33
18	17	3	11
19	18	2	4
20	19	1	3
21	20	0	0

Again we show the initial and final estimates. For convenience, the output is rearranged in a horizontal format. Also, we have omitted the DELTA matrix that contains the item parameters in exponential form. The result is shown in Table 4.

Table 4. Initial and Final Parameter Estimates for the Data from Table 3

ITERATION 0 IMPROVEMENTS	EPSILON	ITERATION 3 IMPROVEMENTS	EPSILON
0.0	-1.6466	-0.0005	-2.0068
0.0	-1.0796	-0.0002	-1.3671
0.0	-0.9895	-0.0002	-1.2626
0.0	-0.6082	-0.0001	0.8102
0.0	0.3330	0.0001	0.3786
0.0	0.3196	0.0001	0.3575
0.0	-0.2787	-0.0000	-0.4054
0.0	-0.1582	-0.0000	-0.2543
0.0	0.8678	0.0002	1.0883
0.0	0.9735	0.0002	1.2295
0.0	-2.3152	-0.0012	-2.7296
0.0	-1.1806	0.0002	-1.4831
0.0	-0.8361	-0.0001	-1.0826
0.0	-0.1336	0.0000	-0.2232
0.0	0.9970	0.0002	1.2610
0.0	1.0787	0.0003	1.3702
0.0	0.9455	0.0002	1.1922
0.0	1.1131	0.0003	1.4162
0.0	1.4578	0.0006	1.8751
	1.1431		1.4562

Subroutine READ3 allows us to carry out the goodness of fit test for the model described in section 4. If we read JTEST = 2 on the first control card the program will call READ3 after READ1 and accordingly estimate the item parameters based on all the subgroups of persons that are indicated on control card 5. The procedure is explained in detail in section 5. For the second example of this section we have chosen the following grouping of individuals:

group 1: scores from 0-9
group 2: scores from 10-11
group 3: scores equal to 12
group 4: scores from 13-14
group 5: scores from 15-20

The complete set of control cards are shown below.

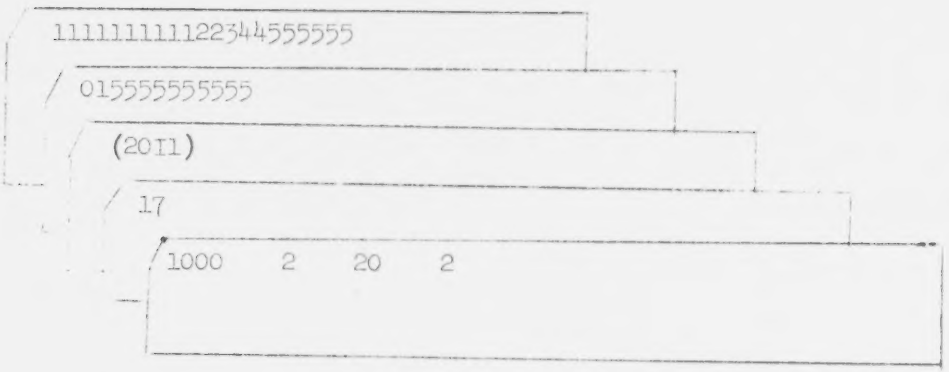


Table 5 shows the final item parameter estimates for all 5 subgroups of individuals.

Table 5. Final Estimates for 5 Subgroups in the SAT Data

Group	1 EPSILON	2 EPSILON	3 EPSILON	4 EPSILON	5 EPSILON
	-2.5856	-1.9100	-1.9337	-1.8065	-1.8272
	-0.7567	-1.2885	-1.6812	-1.5862	-1.2306
	-1.2416	-1.2300	-1.2215	-1.1003	-1.1950
	-1.0057	-0.7968	-0.6398	-0.6662	-0.4299
	0.6630	0.3104	0.4040	0.2065	0.4659
	0.4008	0.4805	0.5064	0.6138	0.2184
	-0.2920	-0.5551	-0.3671	-0.1747	-0.1120
	0.4990	-0.3694	-0.7184	-0.5256	-0.6984
	1.0653	1.5539	1.5204	1.4196	0.7769
	1.1590	2.1044	1.9033	1.1966	1.2049
	-3.9873	-3.0807	-2.5309	-2.4738	-2.4125
	-2.2594	-1.7366	-1.4629	-1.4332	-0.4299
	-1.7484	-1.4784	-0.7979	-0.8266	-0.0519
	-0.3472	-0.1699	0.0805	0.1426	-0.2789
	1.5907	1.0476	1.2308	1.0090	0.9023
	1.6424	1.4006	1.2308	1.1966	0.6639
	1.5598	1.1980	0.7314	0.8463	0.5608
	1.7047	1.4006	1.2308	1.0684	1.0433
	2.2032	1.8236	1.5204	1.1966	1.6249
	1.7360	1.2958	0.9946	1.6969	1.2049

If the model holds we shall find the same parameters apart from random errors.

In subroutine LOGLIK, $-2\log L(\Theta)$, where $L(\Theta)$ is given by (14), is computed with the final parameter estimates as arguments. Hence we can easily compute the test statistic Z given by (13) by calling LOGLIK after each estimation of parameters. Table 6 shows $-2\log L(\Theta)$ with the group estimates inserted and $L(\Theta)$ with the overall estimates inserted.

Table 6. $-2\log L(\Theta)$ with Overall and $-2\log L_g(\Theta)$ with the Group Estimates as Arguments

Overall	15237.40
Group 1	5948.82
Group 2	3181.22
Group 3	1642.60
Group 4	2687.35
Group 5	1510.98

By subtraction we find from Table 6, $z = 266.43$, which with $f = 76$ degrees of freedom is highly significant. This is in accordance with Lord's (1968) results, where both the item discriminating powers and the guessing effect varied from item to item. In the Rasch model the item discriminating powers are assumed to be equal and no compensation for guessing is made.

References

Andersen, E. B. (1967). On partial sufficiency and partial ancillarity. Skandinavisk Aktuarietidskrift, pp. 137-152.

Andersen, E. B. (1969). A computer solution to a set of conditional likelihood equations. Journ. Royal. Stat. Soc. B. (To appear)

Andersen, E. B. (1970a). Asymptotic properties of conditional maximum likelihood estimators. Journ. Royal Stat. Soc. B., vol. 32, pp. 283-301.

Andersen, E. B. (1970b). Conditional inference and multiple choice questionnaires. Technical Report No. 8. Copenhagen School of Economics.

Andersen, E. B. (1971). A goodness of fit test for the Rasch model. Technical Report. Copenhagen School of Economics.

Anderson, J., Kearney, G. E., and Everett, A. V. (1968). An evaluation of Rasch's structural model for test items. Brit. Journ. Math. Stat. Psych., vol. 21, pp. 231-238.

Lehmann, E. L. (1959). Testing Statistical Hypotheses. N.Y.: Wiley.

Lord, F. M. (1968). An analysis of the Verbal Scholastic Aptitude Test using Birnbaum's three-parameter logistic model. Educ. Psych. Meas., vol. 28, pp. 989-1021.

Rasch, G. (1960). Probabilistic Models for Some Intelligence and Attainment Tests. Copenhagen: Danmarks Paedagogiske Institut.

Rasch, G. (1961). On general laws and the meaning of measurement in psychology. Proc. Fourth Berk. Symp. on Math. Stat. and Prob. Vol. IV. Pp. 321-333.

Rasch, G. (1966a). An individualistic approach to item analysis. In P. F. Lazarsfeld and N. W. Henry (Eds.), Readings in Math. Soc. Sciences. Science Research Associates, Chicago. Pp. 89-109.

Rasch, G. (1966b). An item analysis which takes individual differences into account. Brit. Journ. Math. and Stat. Psych., vol. 19, pp. 49-57.

Stene, J. (1967). Einführung in Rasch's Theorie der Psychologischen Messung. In G. Fischer (Ed.), Psychologische Test Theorie. Bern: Huber Verlag.

Appendix

Listing of Main Program and Subroutines

```
FORTRAN IV G LEVEL 18                MAIN              DATE = 72040        12/54/42

C     THIS PROGRAM COMPUTES CONDITIONAL MAXIMUM LIKELIHOOD ESTIMATORS   NR4     1
C     FOR QUESTIONNAIRES. REFERENCE.. E.B.ANDERSEN.. THE NUMERICAL      NR4     2
C     SOLUTION OF A SET OF CONDITIONAL LIKELIHOOD EQUATIONS. TECHNICAL  NR4     3
C     REPORT. COPENHAGEN SCHOOL OF ECONOMICS. 1969.                     NR4     4
0001        DIMENSION X(286),S(286,3)                                   NR4     5
0002        DIMENSION KO(4),IVEC(50)                                    NR4     6
0003        DIMENSION CE(30),CT(30)                                     NR4     7
0004        DIMENSION          C(30,30),CC(30,30)                       NR4     8
0005        DIMENSION DELTA(50,2),EPS(50,2),IA(50,2)                    NR4     9
0006        DIMENSION  N(286)                                           NR4    10
0007        DIMENSION NKO(268)                                          NR4    11
0008        COMMON M,K,NA                                               NR4    12
0009        DOUBLE PRECISION EPS,DC,DF                                  NR4    13
0010        DOUBLE PRECISION DELTA,C,CC                                 NR4    14
0011        DOUBLE PRECISION G,XN                                       NR4    15
0012      1 CONTINUE                                                    NR4    16
0013        READ(5,100) NA,M,K,JTEST
0014      100 FORMAT(4I6)
0015        WRITE(6,100) NA,M,K                                         NR4    19
0016        IF(M.EQ.0) GOTO 2                                           NR4    20
0017        N1=IFIX(PINOM(M-1,K+M-1))                                   NR4    21
0018        N2=M-1                                                      NR4    22
0019        N5=K*(M-1)                                                  NR4    23
0020        N10=(M-1)*(K-1)                                             NR4    24
0021        CALL SUB (IA,K,M,X,N1,DELTA,KO,IVEC,C,CC,N10,N5,DC,AAAAA,PQR,N2,G,NR4    25
          * XN,EPS,CE,CT,JTEST,NA,NKO)
0022        GOTO 1                                                      NR4    27
0023      2 STOP                                                        NR4    28
0024        END                                                         NR4    29
```

```
FORTRAN IV G LEVEL  18                  SUB                DATE = 72040          12/54/42

0001            SUBROUTINE SUB(IA,K,M,N,N1,DELTA,KO,IVEC,C,CC,N10,N5,DC,AAAAA,PQR,NR4
               *N2,G,XX,EPS,CE,CT,JTEST,NA,RKO)
                C     THIS SUBROUTINE CONTROLS THE RUN OF THE PROGRAM. ON THE CALL OF    NR4   32
                C     THE ROUTINE THE PARAMETERS N,K,NA AND JTEST CONTAINS..             NR4   33
                C     K= NUMBER OF ITEMS                                                 NR4   34
                C      = NUMBER OF ANSWERCATEGORIES                                      NR4   35
                C     NA= NUMBER OF PERSONS FOR GENERATED DATA.                          NR4   36
                C     JTEST=-1 GIVES GENERATED DATA.                                     NR4   37
                C     JTEST=0 GIVES INPUT ON COMPACT FORM                                NR4   38
                C     JTEST=+1 GIVES DATA ON INDIVIDUAL FORM                             NR4   39
                C     JTEST=+2 GIVES GROUPED DATA.                                       NR4   40
                C     N1, 2,N5 AND N10 SPECIFIES DIMENSIONS OF MATRICES.                 NR4   41
0002            DIMENSION DELTA(K,K)                                                     NR4   42
0003            DIMENSION C(N1),RKO(N1)                                                  NR4   43
0004            DIMENSION IA(K,M)                                                        NR4   44
0005            DIMENSION KO(N) ,IVEC(K)                                                 NR4   45
0006            DIMENSION              P(4),CC(N10,N10),A(150),F(50),H(360),HH(360),     NR4   46
               * AK(360),AKK(360),D(165),DD(165), C(N5,N5),EPS(K,M)
0007            DIMENSION CE(N5),CT(N5),XORD(16),CMAX(1),CMIN(1),CO(1)                   NR4   48
0008            DIMENSION DF(50)
0009            DIMENSION G(N1,N2),X (N1)                                                NR4   50
0010            DIMENSION IX(4),P(4),THE(50),PO(4),DEL(4)
0011            INTEGER R                                                                NR4   52
0012            DOUBLE PRECISION DFLOAT                                                  NR4   53
0013            DOUBLE PRECISION DELTA,G,CC,A,F,H,HH,AK,AKK,D,DD,C,EPS,TOL,BSUM
0014            DOUBLE PRECISION ALR,XX,S
0015            DOUBLEPRECISION DYY,FQX                                                  NR4   56
0016            DOUBLE PRECISION DF,DC
0017            NVEC=0                                                                   NR4   58
0018            IT=0                                                                     NR4   59
0019            BSUM=0.                                                                  NR4   60
0020            TOL=0.5D-2                                                               NR4   61
0021            ALR=0.                                                                   NR4   62
0022            NU=-1                                                                    NR4   63
0023            NMAX=30                                                                  NR4   64
0024            KC=K-1                                                                   NR4   65
0025            MD=M-1                                                                   NR4   66
0026            M1=M-1                                                                   NR4   67
0027            K1=K-1                                                                   NR4   68
0028          8 FORMAT(1X,4I3,2X,I4,2X,I4)                                               NR4   69
0029         11 FORMAT(20I3)                                                             NR4   70
0030            IND=0                                                                    NR4   71
                C     SPECIFICATION OF INPUT                                             NR4   72
0031            IF(JTEST)32111,32110,32132                                               NR4   73
0032         32111 CALL DATASU (THE,N,IA,IX,DEL,R,P,K,M,NA,N1,RO,IVEC)                   NR4   74
0033            GOTO 1399                                                                NR4   75
0034         32110 CONTINUE                                                              NR4   76
0035            CALL READ2(K,M,IA,N,N1)                                                  NR4   77
0036            CALL WRITE1(N,IA,K,M,N1)                                                 NR4   78
0037            GOTO 1399                                                                NR4   79
0038         32132 CONTINUE                                                              NR4   80
0039           450 IT=0                                                                  NR4   81
0040            DO 1 I=1,K                                                               NR4   82
0041            DO 1 J=1,M                                                               NR4   83
0042          1 DELTA(I,J) = 1.0D+0                                                      NR4   84
0043            DO 298 I=1,N1                                                            NR4   85
0044          298 N(I)=0                                                                 NR4   86
0045            DO 297 I=1,K                                                             NR4   87
```

```
FORTRAN IV G LEVEL  18                  SUB                  DATE = 72040          12/54/42

0046                  DO 297 J=1,M                                                    NR4   88
0047              297 IA(I,J)=0                                                       NR4   89
0048                  NU=IND                                                          NR4   90
0049                  WRITE(6,9937)N1                                                 NR4   91
0050             9937 FORMAT(1X,I4)                                                   NR4   92
0051                  CALL READ1(IVEC,IR,IA,N,K,M,NU,NA,N1)                           NR4   93
0052                  CALL WRITE1 (N,IA,K,M,N1)                                       NR4   94
0053             1399 CONTINUE                                                        NR4   95
0054                  IF(JTEST.LT.0)JTEST=-JTEST                                      NR4   96
0055                  N3=M                                                            NR4   97
0056                  N4=K                                                            NR4   98
0057                  N6=IFIX(BINOM(M-2,K-(K+1)/(M-1)+M-2))*((K+1)/(M-1)+1)           NR4   99
0058                  N7=IFIX(BINOM(M-2,K-2+M-2))                                     NR4  100
0059                  N8=IFIX(BINOM(M-2,K-1+M-2))                                     NR4  101
0060                  N9=IFIX(BINOM(M-1,K+M-2))                                       NR4  102
0061                  IF(JTEST-1)452,452,453                                          NR4  103
0062              453 READ(5,454)NKO                                                  NR4  104
0063                  IND=1                                                           NR4  105
0064                  JND=0                                                           NR4  106
0065              455 JND=JND+1                                                       NR4  107
0066              454 FORMAT(40I2)                                                    NR4  108
0067                  IF (JND.EQ.6)RETURN                                             NR4  109
0068                  CALL READ3(JND,N1,K,M,IA,N,NKO,IVEC)                            NR4  110
0069                  WRITE(6,1440)JND                                                NR4  111
0070             1440 FORMAT(1H1,12HGROUPING NO.,I4)                                  NR4  112
0071                  CALL WRITE1(N,IA,K,M,N1)                                        NR4  113
0072              452 DO 124 I=1,N5                                                   NR4  114
0073              124 F(I)=0.                                                         NR4  115
                C       COMPUTATION OF INITIAL VALUES                               NR4  116
0074                  CALL INTIAL(IA,EPS,K,M,IER)                                     NR4  117
0075                  IF(IER)123,123,130
0076              123 CONTINUE                                                        NR4  119
                C       ITERATIONS START                                            NR4  120
0077                  IT=0                                                            NR4  121
0078               99 WRITE(6,999)IT                                                  NR4  122
0079              999 FORMAT(10H1ITERATION,I3)                                        NR4  123
0080                  DO 220 I=1,KC                                                   NR4  124
0081                  DO 220 L=1,M1                                                   NR4  125
0082                  IP=(I-1)*(M-1)+L                                                NR4  126
0083                  EPS(I,L)=EPS(I,L)+F(IP)                                         NR4  127
0084              220 DELTA(I,L)=DEXP(EPS(I,L))                                       NR4  128
0085                  DO 222 L=1,M1                                                   NR4  129
0086                  S=0.                                                            NR4  130
0087                  DO 221 I=1,KC                                                   NR4  131
0088              221 S=S+EPS(I,L)                                                    NR4  132
0089                  EPS(K,L)=-S                                                     NR4  133
0090              222 DELTA(K,L)*DEXP(EPS(K,L))                                       NR4  134
0091                  CALL WRITE2(F,DELTA,EPS,K,M,M1,KC)                              NR4  135
0092               73 IF( IT.EQ. 6) GO TO 130                                         NR4  136
                C       TEST OF ACCURACY                                            NR4  137
0093                  IF( IT.EQ.0)GOTO 3                                              NR4  138
0094                  DO 74 J=1,N10                                                   NR4  139
0095                  FQX=DABS(F(J))                                                  NR4  140
0096                  IF(FQX.GE.TOL) GOTO 3                                           NR4  141
0097               74 CONTINUE                                                        NR4  142
0098                  GOTO 130                                                        NR4  143
0099                3 CONTINUE                                                        NR4  144
                C       COMPUTATION OF  COEFFICIENTMATRICES                         NR4  145
```

```
FORTRAN IV G LEVEL 18                    SUB                    DATE = 72040          12/54/42

0108                CALL ESTIM(N,G,IA,B,DELTA,AM,F,H,MM,AK,C,D,DD,%1,%2,%3,%4,%5,%6,    VR4  146
                   * %7,%8,%9,V,AK%,CC,%10)                                              VR4  147
0101                IT=IT+1                                                              VR4  148
0102                DO 754 J=1,M10                                                       VR4  149
0103                DO 754 I=1,M10                                                       VR4  150
0104          754   CC(J,I)=-CC(J,I)                                                     VR4  151
              C     SOLUTION OF LIKELIHOOD EQUATIONS                                     VR4  152
0105                CALL RSIMQ (CC,DF, 10,%5,%15,C,F)                                    VR4  153
              C     COMPUTATION OF LOG-LIKELIHOOD RATIO                                  VR4  154
0106                CALL LOGLIK (G,IA,EPS,N,M,%1,ALR,X%%,%2)                             VR4  155
0107                GO TO 99                                                             VR4  156
0108          190   WRITE(6,198)ALR                                                      VR4  157
0109          198   FORMAT(1H1,7HLOGLIKE,2F10.4)                                         VR4  158
0110                IF(IT.EQ.1)GOTO 455                                                  VR4  159
0111                RETURN                                                               VR4  160
0112                END                                                                  VR4  161
```

```
FORTRAN IV G LEVEL 18                    INTIAL                 DATE = 72040          12/54/42

0001                SUBROUTINE INTIAL (IA,EPS,K,M,IER)                                   VR4  162
0002                DIMENSION IA(K,M),EPS(K,M)                                           VR4  163
0003                DOUBLE PRECISION PSUM,EPS                                            VR4  164
0004                DOUBLE PRECISION DFLOAT                                              VR4  165
0005                IER=0                                                                VR4  166
0006                DO 76 J=1,M                                                          VR4  167
0007                DO 76 I=1,K
0008                IF(IA(I,J).LE.0)IER=1
0009          76    CONTINUE                                                             VR4  170
0010                IF(IER.EQ.1) RETURN                                                  VR4  171
0011                DO 75 J=1,M                                                          VR4  172
0012                PSUM=0.                                                              VR4  173
0013                DO 71 I=1,K                                                          VR4  174
0014          71    PSUM=PSUM+DLOG(DFLOAT(IA(I,J)))-DLOG(DFLOAT(IA(I,M)))                VR4  175
0015                DO 75 I=1,K                                                          VR4  176
0016          75    EPS(I,J)=DLOG(DFLOAT(IA(I,J)))-DLOG(DFLOAT(IA(I,M)))-PSUM/DFLOAT(K)VR4  177
                   * )
0017                RETURN                                                               VR4  178
0018                END                                                                  VR4  179
                                                                                         VR4  180
```

```
FORTRAN IV G LEVEL 18                READ1              DATE = 72040        12/54/42

0001            SUBROUTINE READ1 (IVEC,IR,IA,N,K,M,NU,NA,N1)                        NR5  293
                C     READS INDIVIDUAL DATA FROM TAPE NU. NU AND THE FORMAT FM MUST  NR5  294
                C     APPEAR FIRST IN DATA CARDS WITH FORMATS (I5) AND (80A1). EACH  NR5  295
                C     PERSONS DATA ARE STORED ON TAPE 98 TOGETHER WITH THE INDEX OF  NR5  296
                C     THE SCOREVECTORINDEX. IA AND N CONTAIN ITEM AND PERSONTOTALS ON NR5  297
                C     RETURN. M,K AND N1 ARE DIMENSIONS OF MATRICES.                 NR5  298
                C      THE MATRIX IB CONTAINS THE CODES OF THE ANSWER CATEGORIES. THUS NR5  299
                C     ROW L OF IB MUST CONTAIN ALL INTEGERS ON THE INPUT TAPE NU, THAT NR5  300
                C     SHOULD BE CODED AS L. SINCE THE NUMBER OF COLUMNS IN IB IS 3,  NR5  301
                C     AS MANY AS 3 DIFFERENT INTEGERS CAN BE CODED AS THE SAME NUMBER. NR5  302
0002                  DIMENSION IVEC(K),IR(M),IA(K,M),N(N1)                          NR5  303
0003                  DIMENSION IB(4,3),IAA(50)                                      NR5  304
0004                  DIMENSION FM(60)                                               NR5  305
0005                  REWIND 1                                                       NR5  306
0006                  NR=NA                                                          NR5  307
0007                  READ(5,100)NU                                                  NR5  308
0008                  READ(5,2)FM                                                    NR5  309
0009                  READ(5,1010)IB                                                 NR5  310
0010             1010 FORMAT(4012)                                                   NR5  311
0011                2 FORMAT(60A1)                                                  NR5  312
0012              100 FORMAT(I2)                                                    NR5  313
                  299 READ(NU,FM)(IAA(I),I=1,K)
0013                  DO 1 I=1,M                                                     NR5  315
0014                1 IR(I)=0                                                      NR5  316
0015                  NR=NR-1                                                        NR5  317
0016                  IF(NR) 295,295,291                                             NR5  318
0017              291 DO 1011 I=1,K                                                  NR5  319
0018                  DO 1011 J=1,3                                                  NR5  320
0019                  DO 1011 L=1,M                                                  NR5  321
0020                  IF(IAA(I).GT.M) GOTO 299                                       NR5  322
0021             1011 IF(IAA(I).EQ.IB(L,J))IVEC(I)=L                                 NR5  323
0022                  DO 303 I=1,K                                                   NR5  324
0023                  DO 303 L=1,M                                                   NR5  325
0024                  IF(IVEC(I).EQ.L  )IR(L)=IR(L)+1                                NR5  326
0025              303 IF(IVEC(I).EQ.L  ) IA(I,L)=IA(I,L)+1                           NR5  327
0026                  I=INDEX(IR,0,M)                                                NR5  328
0027                  N(I)=N(I)+1                                                    NR5  329
0028                  WRITE(6,10)NR,IVEC,I                                           NR5  330
0029               10 FORMAT(1X,I4,9I3)                                              NR5  331
0030                  WRITE(98)NR,IVEC,I                                             NR5  332
0031                  GOTO  299                                                      NR5  333
0032              295 CONTINUE                                                       NR5  334
0033                  WRITE(98)NR,IVEC,I                                             NR5  335
0034                  RETURN                                                         NR5  336
0035                  END                                                            NR5  337
```

```
FORTRAN IV G LEVEL  18                  READ2            DATE = 72040          12/54/42

0001                  SUBROUTINE READ2(K,M,IA,N,N1)                                  NR5  308
                C     READ PERSONTOTALS N AND      ITEMTOTALS IA ON COMPACT FORM WITH NR5  309
                C     FORMAT(20I4). K,M AND N1 SPECIFIES DIMENSIONS OF MATRICES.     NR5  310
0002                  DIMENSION N(N1),IA(K,M)                                        NR5  311
0003                  READ(5,1)N                                                     NR5  312
0004                  DO 3 I=1,K                                                     NR5  313
0005                3 READ(5,1)(IA(I,L),L=1,M)                                       NR5  314
0006                1 FORMAT(20I4)                                                   NR5  315
0007                  RETURN                                                         NR5  316
0008                  END                                                            NR5  317
```

```
FORTRAN IV G LEVEL  18                  READ3            DATE = 72040          12/54/42

0001                  SUBROUTINE READ3(JND,N1,K,M,IA,N,NKO,IVEC)                     NR5  318
                C     READ DATA IN GROUPED FORM. JND IS GROUPINDICATOR, NKO HAS SAME NR5  319
                C     LENGTH AS N     READ3 INCLUDES ONLY PERSONS THAT BELONG TO A COM- NR5  320
                C     PONENT OF N FOR WHICH THE CORRESPONDING COMPONENT IN NKO HAS THE NR5  321
                C     VALUE JND. ALL PERSONS MUST BE READABLE ON TAPE 98. IA AND N   NR5  322
                C     CONTAIN ON RETURN ITEM AND PERSONTOTALS. K,M AND N1 SPECIFIES  NR5  323
                C     DIMENSIONS OF MATRICES.                                        NR5  324
0002                  DIMENSION IVEC(K)                                              NR5  325
0003                  DIMENSION IA(K,M),N(N1),NKO(125)                               NR5  326
0004                  DO 1 I=1,N1                                                    NR5  327
0005                1 N(I)=0                                                         NR5  328
0006                  DO 2 I=1,K                                                     NR5  329
0007                  DO 2 L=1,M                                                     NR5  330
0008                2 IA(I,L)=0                                                      NR5  331
0009                  REWIND 1                                                       NR5  332
0010                3 READ(98)NR,IVEC,I                                              NR5  333
0011                  IF(NR.LE.0)GOTO 10                                             NR5  334
0012                  IF(NKO(I).NE.JND)GOTO 3                                        NR5  335
0013                  DO 5 J=1,K                                                     NR5  336
0014                  DO 5 L=1,M                                                     NR5  337
0015                5 IF(IVEC(J).EQ.L  )IA(J,L)=IA(J,L)+1                            NR5  338
0016                  N(I)=N(I)+1                                                    NR5  339
0017                  GOTO 3                                                         NR5  340
0018               10 RETURN                                                         NR5  341
0019                  END                                                            NR5  342
```

```
FORTRAN IV G LEVEL  18                  DATASU          DATE = 72040        12/54/42

0001                  SUBROUTINE DATASU (THE,NR,IA,IX,DEL,IR,P,K,M,N,N1,RO,IVEC)          NR5    1
                C     GENERATES DATA FOR NA RANDOMLY SELECTED PERSONS FOR GIVEN PARAME-    NR5    2
                C     TERS THE(K),DEL(M) AND RO(M). PARAMETERS ARE READ IN THIS ORDER      NR5    3
                C     WITH FORMAT(10F8.3). IA IS RESULTANT ITEMTOTALS AND NP RESULTANT     NR5    4
                C     PERSONTOTALS.K,M,N AND N1 SPECIFIES DIMENSION OF MATRICES.           NR5    5
0002                  DIMENSION THE(K   ),NR(N1),IA(K,M),IX(M),IR(M),P(M),DEL(M)          NR5    6
0003                  DIMENSION RO(M),IVEC(K)                                             NR5    7
0004                  READ(.,1) THE                                                       NR5    8
0005                  READ(5,1) DEL                                                       NR5    9
0006                  READ(5,1)RO                                                         NR5   10
0007                1 FORMAT(10F8.3)                                                      NR5   11
0008                  WRITE(6,1) THE                                                      NR5   12
0009                  WRITE(6,1)DEL                                                       NR5   13
0010                  WRITE(6,1) RO                                                       NR5   14
0011                  DO 13 I=1,M                                                         NR5   15
0012                  DO 13 J=1,K                                                         NR5   16
0013               13 IA(J,I)=0                                                           NR5   17
0014                  DO 14 I=1,N1                                                        NR5   18
0015               14 NR(I)=0                                                             NR5   19
0016                  J=236421                                                            NR5   20
0017                  DO 4 I=1,N                                                          NR5   21
0018                  CALL GAUSS(J,1.,0.,U)                                               NR5   22
0019                3 FORMAT(1X,5HPAR.=,F8.3)                                             NR5   23
0020                  DO 15 L=1,M                                                         NR5   24
0021               15 IR(L)=0                                                             NR5   25
0022                  DO 7 JJ=1,K                                                         NR5   26
0023                  S=0.                                                                NR5   27
0024                  DO 5 L=1,M                                                          NR5   28
0025                  P(L)=EXP((THE(JJ   )+U)*DEL(L)+RO(L))                               NR5   29
0026                5 S=S+P(L)                                                            NR5   30
0027                  DO 6 L=1,M                                                          NR5   31
0028                6 P(L)=P(L)/S                                                         NR5   32
0029                  CALL PROB (IY,P,M,J )                                               NR5   33
0030                  DO 7 L=1,M                                                          NR5   34
0031                  IF(IY-L)7,8,7                                                       NR5   35
0032                8 IA(JJ,L)=IA(JJ,L)+1                                                 NR5   36
0033                  IR(L)=IR(L)+1                                                       NR5   37
0034                  IVEC(JJ)=L                                                          NR5   38
0035                7 CONTINUE                                                            NR5   39
0036                  IQ=INDEX(IR,0,M)                                                    NR5   40
0037                  NR(IQ)=NR(IQ)+1                                                     NR5   41
0038                  WRITE(6,10) I,IVEC,IQ                                               NR5   42
0039                4 CONTINUE                                                            NR5   43
0040                  I=0                                                                 NR5   44
0041                  WRITE(6,11) I,IVEC,IQ                                               NR5   45
0042                  CALL WRITE1(NR,IA,K,M,N1)                                           NR5   46
0043                9 FORMAT(1X,4I4)                                                      NR5   47
0044               10 FORMAT(15H QUESTIONMATRIX   )                                       NR5   48
0045               11 FORMAT(13H PERSONTOTALS   )                                         NR5   49
0046                  RETURN                                                              NR5   50
0047                  END                                                                 NR5   51
```

```
FORTRAN IV G LEVEL 18                    WRITE1          DATE = 72040      12/54/42

0001            SUBROUTINE WRITE1 (N,IA,K,M,N1)                                  NR5   52
C               WRITE PERSONTOTALS N AND ITEMTOTALS IA, K,M AND N1 SPECIFIES     NR5   53
C               DIMENSIONS OF MATRICES.
0002            DIMENSION N(N1),IA(K,M),IR(4)                                    NR5   54
0003            KEY=1                                                            NR5   55
0004            WRITE(6,304)                                                     NR5   56
0005          304 FORMAT(1H1,12HPERSONTOTALS)                                    NR5   57
0006            I=1                                                              NR5   58
0007            M1=M-1                                                           NR5   59
0008            DO 1 L=1,M1                                                      NR5   60
0009          1 IR(L)=0                                                          NR5   61
0010            IR(M)=K                                                          NR5   62
0011          305 WRITE(6,306)I,IR,N(I)                                          NR5   63
0012            IF(IR(1).EQ.K) GOTO 2                                            NR5   64
0013            CALL VEC(IR,KEY,M)                                               NR5   65
0014            I=I+1                                                            NR5   66
0015            GOTO 305                                                         NR5   67
0016          306 FORMAT(1X,I3,3X,5I5)                                           NR5   68
0017            2 CONTINUE                                                       NR5   69
0018            WRITE(6,307)                                                     NR5   70
0019          307 FORMAT(1H1,14HQUESTIONTOTALS)                                  NR5   71
0020            DO 308 I=1,K                                                     NR5   72
0021          308 WRITE(6,309)I,(IA(I,J),J=1,M)                                  NR5   73
0022          309 FORMAT(1X,I2,3X,5I5)                                           NR5   74
0023            RETURN                                                           NR5   75
0024            END                                                              NR5   76
                                                                                 NR5   77
```

```
FORTRAN IV G LEVEL 18                    WRITE2          DATE = 72040      12/54/42

0001            SUBROUTINE WRITE2(F,DELTA,EPS,K,M,M1,KC)                         NR5   78
0002            DIMENSION EPS(K,M),DELTA(K,M),F(286),FF(10,4)                    NR5   79
0003            DOUBLE PRECISION EPS,DELTA,F                                     NR5   80
0004            WRITE(6,998)                                                     NR5   81
0005          998 FORMAT(1X,12HIMPROVEMENTS)                                     NR5   82
0006            DO 222 I=1,KC                                                    NR5   83
0007            DO 223 L=1,M1                                                    NR5   84
0008            K10=(I-1)*M1+L                                                   NR5   85
0009          223 FF(I,L)=F(K10)                                                 NR5   86
0010          222 WRITE(6,420)(FF(I ,L),L=1,M1)                                  NR5   87
0011          447 WRITE(6,996)                                                   NR5   88
0012          996 FORMAT(6HODELTA)                                               NR5   89
0013            DO 221 I=1,K                                                     NR5   90
0014          221 WRITE(6,420)(DELTA(I,L),L=1,M)                                 NR5   91
0015            WRITE(6,421)                                                     NR5   92
0016          421 FORMAT(9H EPSILON    )                                         NR5   93
0017            DO 422 I=1,K                                                     NR5   94
0018            WRITE(6,420) (EPS(I,J),J=1,M)                                    NR5   95
0019          420 FORMAT(1X,8F8.4)                                               NR5   96
0020          422 CONTINUE                                                       NR5   97
0021            RETURN                                                           NR5   98
0022            END                                                              NR5   99
```

```
FORTRAN IV G LEVEL 18                ESTIM              DATE = 72040          12/54/42

0001                  SUBROUTINE ESTIM(N,G,IA,R,DELTA,AV,F,H,HH,AK,C,D,DD,N1,N2,N3,N4,   NR4  291
                     * N5,N6,N7,N8,N9,V,AKK ,CC,N10)                                     NR4  292
                C     COMPUTES COEFFICIENT MATRIX CC AND RIGHT HAND SIDE OF LIKELIHOOD   NR4  293
                C     EQUATIONS. ON RETURN G CONTAINS GAMMA-RATIOS TO BE USED IN SUB-    NR4  294
                C     ROUTINE LOGLIKE. IA AND N ARE ITEM- AND PERSONTOTALS. DELTA IS     NR4  295
                C     MATRIX OF ITEMPARAMETERS ON EXPONENTIAL FORM. V,K AND N1 THROUGH   NR4  296
                C     N10 SPECIFIES DIMENSIONS OF MATRICES. THE STEPS OF THIS ROUTINE    NR4  297
                C     ARE INDICATED BY COMMENT STATEMENTS.                               NR4  298
0002                  DIMENSION N(N1),G(N1,N2),IA(N4,N3),R(N3),DELTA(N4,N3),AV(N4),F(N5) NR4  299
                     * ,H(N6),HH(N6),AK(N8),AKK(N6),D(N8),DD(N6),C(N5,N5),CC(N10,N10)    NR4  300
0003                  COMMON MG,K,NA                                                     NR4  301
0004                  INTEGER R                                                          NR4  302
0005                  DOUBLE PRECISION G,AKK,DELTA,AV,F,H,HH,AK,D,DD,C,CC,PG             NR4  303
0006                  DOUBLE PRECISION DO,D1                                             NR4  304
0007                  DO=0.0D0                                                           NR4  305
0008                  D1=1.0D0                                                           NR4  306
0009                  NV=K*(V-1)                                                         NR4  307
0010                  DO 50 I=1,NV                                                       NR4  308
0011                  F(I)=DO                                                            NR4  309
0012                  DO 50 J=1,NV                                                       NR4  310
0013                  C(I,J)=DO                                                          NR4  311
0014               50 CONTINUE                                                           NR4  312
                C                                                                        NR4  313
                C     FIRST R-RUN BEGINS                                                 NR4  314
                C     COMPUTES BASIC GAMMARATIOS                                         NR4  315
                C     STORING IN G, WHILE H, HH AND AV ARE AUXILLARY                     NR4  316
                C                                                                        NR4  317
0015                  IC=K-1                                                             NR4  318
0016                  K1=K+1                                                             NR4  319
0017                  DO 51 I=1,K1                                                       NR4  320
0018                  HH(I)=D1                                                           NR4  321
0019               51 CONTINUE                                                           NR4  322
0020                  AV(K)=D1                                                           NR4  323
0021                  R(V)=K-1                                                           NR4  324
0022                  R(V-1)=1                                                           NR4  325
0023                  IF(V.EQ.2)GOTO 905                                                 NR4  326
0024                  NV=V-2                                                             NR4  327
0025                  DO 52 L=1,NV                                                       NR4  328
0026                  R(L)=0                                                             NR4  329
0027               52 CONTINUE                                                           NR4  330
0028              905 CONTINUE                                                           NR4  331
0029                  KEY=2                                                              NR4  332
0030              106 IF(IC.GT.0) GO TO 213                                              NR4  333
0031                  IQ=INDEX(R,0,V-1)                                                  NR4  334
0032                  IF(V.EQ.2)IQ=1                                                     NR4  335
0033                  GO TO 212                                                          NR4  336
0034              213 IQ=INDEX(R,IC,V)                                                   NR4  337
0035              212 I=IC+1                                                             NR4  338
0036              202 CONTINUE                                                           NR4  339
0037              103 IP=(IQ-1)*(IC+1)+I                                                 NR4  340
0038                  H(IP)=DO                                                           NR4  341
0039                  NVV=V-1                                                            NR4  342
0040                  DO 53 J=1,NVV                                                      NR4  343
0041                  R(J)=R(J)-1                                                        NR4  344
0042                  IF(R(J).LT.0) GO TO 100                                            NR4  345
0043                  IQQ=INDEX(R,IC+1,V)                                                NR4  346
0044                  IPP=(IQQ-1)*(IC+2)+I+1                                             NR4  347
0045                  ICC=IC+1                                                           NR4  348
```

```
FORTRAN IV G LEVEL  18                  ESTIM                   DATE = 72040        12/54/42

0046                IB=I                                                                    NR4  349
0047                H(IP)=H(IP)+HH(IPP)*DELTA(IB,J)/AM(ICC)                                 NR4  350
0048             100 R(J)=R(J)+1                                                            NR4  351
0049              53 CONTINUE                                                               NR4  352
0050                IF(I.GE.IC+1) GO TO 101                                                 NR4  353
0051                H(IP)=H(IP)+H(IP+1)                                                     NR4  354
0052             101 I=I-1                                                                  NR4  355
0053                IF(I.LT.1) GO TO 102                                                    NR4  356
0054                GO TO 103                                                               NR4  357
0055             102 NVN=V-1                                                                NR4  358
0056                 IQQ=INDEX(P,0,V)                                                       NR4  359
0057                 DO 54 L=1,NVN                                                          NR4  360
0058                 R(L)=R(L)-1                                                            NR4  361
0059                 IP=(IQ-1)*(IC+1)+1                                                     NR4  362
0060                 IF(R(L).LT.0) GO TO 104                                                NR4  363
0061                 IQA=INDEX(R,IC+1,V)                                                    NR4  364
0062                 IPP=(IQA-1)*(IC+2)+1                                                   NR4  365
0063                 ICI=IC+1                                                               NR4  366
0064                 G(IQQ,L)=HH(IPP)/(AM(ICI)*H(IP))                                       NR4  367
0065             104 R(L)=R(L)+1                                                            NR4  368
0066              54 CONTINUE                                                               NR4  369
0067                 IF(R(1).NE.(K-IC)) GO TO 106                                           NR4  370
0068                 P=BINOM(V-2,V+K-IC-2)                                                  NR4  371
0069                 IP=P                                                                   NR4  372
0070                 IP=IP*(IC+1)                                                           NR4  373
0071                 DO 55 I = 1,IP                                                         NR4  374
0072              55 HH(I)=H(I)                                                             NR4  375
0073                 IF(K.GT.5) GOTO1310                                                    NR4  376
0074                 WRITE(6,1307)R,H                                                       NR4  377
0075            1310 CONTINUE                                                               NR4  378
0076             204 CONTINUE                                                               NR4  379
0077                 IF(IC.EQ.0) GO TO 106                                                  NR4  380
0078                 AM(IC)=DO                                                              NR4  381
0079                 DO 56 I = 1,IQ                                                         NR4  382
0080                 IP=(I-1)*(IC+1)+1                                                      NR4  383
0081                 IF(H(IP).LE.AM(IC)) GO TO 56                                           NR4  384
0082                 AM(IC)=H(IP)                                                           NR4  385
0083              56 CONTINUE                                                               NR4  386
0084                 IC=IC-1                                                                NR4  387
0085             106 CALL VEC(R,KEY,M)                                                      NR4  388
0086                 IF(KEY.LT.3) GO TO 105                                                 NR4  389
0087            1311 CONTINUE                                                               NR4  390
0088                 DO 404 I=1,N1
0089             404 WRITE(6,1308)(G(I,L),L=1,3)
                C
                C    FIRST R-RUN IS TERMINATED.                                             NR4  391
                C    SECOND R-RUN BEGINS.                                                   NR4  392
                C    COMPUTES GAMMARATIOS                                                   NR4  393
                C    IN H, AK AND D, WHILE                                                  NR4  394
                C    HH, AKK AND DD ARE AUXILLARY.                                          NR4  395
                C                                                                           NR4  396
0090              30 CONTINUE                                                               NR4  397
0091             250 CONTINUE                                                               NR4  398
0092                 KH=K/2                                                                 NR4  399
0093                 DO 57 I = 1,K                                                          NR4  400
0094                 IF(K-I)133,133,131                                                     NR4  401
0095             131  NB=I+1                                                               NR4  402
0096                 GO TO 134                                                              NR4  403
                                                                                            NR4  404
```

```
FORTRAN IV G LEVEL 18                    ESTIM                    DATE = 72040          12/54/42

0097          133 NB=K                                                                    NR4  405
0098          134 DO 63 IH=NB,K                                                           NR4  406
0099              IC=2                                                                    NR4  407
0100              H(1)=D1                                                                 NR4  408
0101              AK(1)=D1                                                                NR4  409
0102              D(1)=D1                                                                 NR4  410
0103              R(M)=K                                                                  NR4  411
0104              IC=K                                                                    NR4  412
0105              NMM=M-1                                                                 NR4  413
0106              DO 58 L=1,NMM                                                           NR4  414
0107           58 R(L)=0                                                                  NR4  415
0108              IQ=1                                                                    NR4  416
0109              IQP=1                                                                   NR4  417
0110              GO TO 107                                                               NR4  418
0111          108 IQ=INDEX(R,IC,M)                                                        NR4  419
0112              IQP=INDEX(R,0,M)                                                        NR4  420
0113              H(IQ)=DO                                                                NR4  421
0114              AK(IQ)=DO                                                               NR4  422
0115              D(IQ)=DO                                                                NR4  423
0116              NVA = M-1                                                               NR4  424
0117              IF(ICO.EQ.1) GOTO 1320                                                  NR4  425
0118              IF(IC-K)1320,1321,1321                                                  NR4  426
0119         1321 CONTINUE                                                                NR4  427
0120              DO 59 L = 1,NVA                                                         NR4  428
0121              R(L)=R(L)-1                                                             NR4  429
0122              IF(R(L).LT.0) GO TO 116                                                 NR4  430
0123              IQQ=INDEX(R,IC+1,M)                                                     NR4  431
0124              H(IQ)=H(IQ)+HH(IQQ)*G(IQP,L)*DELTA(I,L)                                 NR4  432
0125              AK(IQ)=AK(IQ)+AKK(IQQ)*G(IQP,L)*DELTA(IH,L)                             NR4  433
0126              D(IQ)=D(IQ)+DD(IQQ)*G(IQP,L)*DELTA(IH,L)                                NR4  434
0127           116 R(L)=R(L)+1                                                            NR4  435
0128            59 CONTINUE                                                               NR4  436
0129              H(IQ)=D1-H(IQ)                                                          NR4  437
0130              AK(IQ)=D1-AK(IQ)                                                        NR4  438
0131              IF(I.EQ.K) GO TO 311                                                    NR4  439
0132              D(IQ)=H(IQ)-D(IQ)                                                       NR4  440
0133              GOTO 1360                                                               NR4  441
0134         1320 CONTINUE                                                                NR4  442
0135              R(1)=R(1)+1                                                             NR4  443
0136              IQQ=INDEX(R,IC-1,M)                                                     NR4  444
0137              IF(IC.EQ.1)IQQ=INDEX(R,0,M-1)                                           NR4  445
0138              R(M)=R(M)-1                                                             NR4  446
0139              IPQ=INDEX(R,0,M)                                                        NR4  447
0140              R(M)=R(M)+1                                                             NR4  448
0141               H(IQ)=H(IQ)+(D1-HH(IQQ))/G(IPQ,1)/DELTA(I,1)                           NR4  449
0142              AK(IQ)=AK(IQ)+(D1-AKK(IQQ))/G(IPQ,1)/DELTA(IH,1)                        NR4  450
0143              IF(I.EQ.K) GOTO 6668                                                    NR4  451
0144               D(IQ)=D(IQ)+(HH(IQQ)-DD(IQQ))/G(IPQ,1)/DELTA(IH,1)                     NR4  452
0145         6668 CONTINUE                                                                NR4  453
0146              IF(IC.EQ.1)D(IQ)=DO                                                     NR4  454
0147              IF(M.EQ.2)GOTO 1326                                                     NR4  455
0148              M1=M-1                                                                  NR4  456
0149              DO 1359 L=2,M1                                                          NR4  457
0150              R(L)=R(L)-1                                                             NR4  458
0151              IF(R(L).LT.0)GOTO 1327                                                  NR4  459
0152              IQX=INDEX(R,IC,M)                                                       NR4  460
0153               H(IQ)= H(IQ)-H(IQX)*DELTA(I ,L)/G(IPQ,1)*G(IPQ,L)/DELTA(I ,1)         NR4  461
0154              AK(IQ)=AK(IQ)-AK(IQX)*DELTA(IH,L)/G(IPQ,1)*G(IPQ,L)/DELTA(IH,1)       NR4  462
```

```
FORTRAN IV G LEVEL  18                  ESTIM              DATE = 72040          12/54/42

0155                D(IQ)= D(IQ)- D(IQX)*DELTA(IH,L)/G(IPQ,1)*G(IPQ,L)/DELTA(IH,1)    NR4  463
0156          1327 R(L)=R(L)+1                                                        NR4  464
0157          1359 CONTINUE                                                           NR4  465
0158          1326 R(1)=R(1)-1                                                        NR4  466
0159          1360 CONTINUE                                                           NR4  467
0160               IF(ICD.EQ.1) GOTO 1118                                             NR4  468
                C                                                                    NR4  469
                C   COMPUTED F-VECTOR AND C-MATRIX ARE COEFFICIENTS
                C   OF EQUATIONSYSTEM FOR IMPROVING OF DELTA-VALUES.                 NR4  471
                C                                                                    NR4  472
0161            107 CONTINUE                                                         NR4  473
0162                IF(I.EQ.K) GOTO 311                                              NR4  474
0163                IF(IH.NE.I+1) GO TO 62                                           NR4  475
0164            311 NMB=M-1                                                          NR4  476
0165                R(M)=R(M)-1                                                      NR4  477
0166                DO 60 L=1,NMB                                                    NR4  478
0167                R(L)=R(L)+1                                                      NR4  479
0168                IQQ=INDEX(R,0,M)                                                 NR4  480
0169                IP=(I-1)*(M-1)+L                                                 NR4  481
0170                F(IP)=F(IP)+ FLOAT(N(IQQ))*H(IQ)*G(IQQ,L)                        NR4  482
0171                C(IP,IP)=C(IP,IP)+ FLOAT(N(IQQ))*H(IQ)*G(IQQ,L)*(H(IQ)*G(IQQ,L)-NR4  483
                   * D1/DELTA                                                        NR4  484
                   * (I,L))                                                         NR4  485
0172             31 CONTINUE                                                         NR4  486
0173                NA=L-1                                                           NR4  487
0174                L1=1                                                             NR4  488
0175                IF(KH.GT.IC)NA=M-1                                               NR4  489
0176                IF(KH.GT.IC)L1=L+1                                               NR4  490
0177                IF(NA.LT.L1)GOTO 171                                             NR4  491
0178                DO 61 LL=L1,NA                                                   NR4  492
0179                R(LL)=R(LL)-1                                                    NR4  493
0180                IF(R(LL).LT.0) GO TO 109                                         NR4  494
0181                IQQQ=INDEX(R,IC,M)                                               NR4  495
0182                IPP=(I-1)*(M-1)+LL                                               NR4  496
0183                C(IPP,IP)=C(IP,IPP)+ FLOAT(N(IQQ))*H(IQ)*G(IQQ,L)*H(IQQQ)*      NR4  497
                   * G(IQQ,LL)                                                       NR4  498
0184                C(IP,IPP)=C(IPP,IP)                                              NR4  499
0185            109 R(LL)=R(LL)+1                                                    NR4  500
0186             61 CONTINUE                                                         NR4  501
0187            171 R(L)=R(L)-1                                                      NR4  502
0188             60 CONTINUE                                                         NR4  503
0189                R(M)=R(M)+1                                                      NR4  504
0190                IF(I.EQ.K) GO TO 132                                             NR4  505
0191             62 MH=M-1                                                           NR4  506
0192                R(M)=R(M)-1                                                      NR4  507
0193                DO 68 L=1,MH                                                     NR4  508
0194                IP=(I-1)*(M-1)+L                                                 NR4  509
0195                IPQ=(IH-1)*(M-1)+L                                               NR4  510
0196                R(L)=R(L)+1                                                      NR4  511
0197                IQQ=INDEX(R,0,M)                                                 NR4  512
0198                L1=1                                                             NR4  513
0199                NA=L                                                             NR4  514
0200                IF(KH.GT.IC)L1=L                                                 NR4  515
0201                IF(KH.GT.IC)NA=MH                                                NR4  516
0202                DO 69 LL=L1,NA                                                   NR4  517
0203                IPP=(IH-1)*(M-1)+LL                                              NR4  518
0204                IPPQ=(I-1)*(M-1)+LL                                              NR4  519
0205                R(LL)=R(LL)+1                                                    NR4  520
```

```
FORTRAN IV G LEVEL  18                  ESTIM                DATE = 72040          12/54/42

0206              R(M)=R(M)-1                                                          NR4  521
0207              IF(R(LL).GT.K) GO TO 113                                             NR4  522
0208              IF(IC.LT.2) GO TO 113                                                NR4  523
0209              IQQQ=INDEX(R,0,M)                                                    NR4  524
0210              C(IPP,IP)=C(IP,IPP)- FLOAT(N(IQQQ))*D(IQ)*G(IQQ,L)*G(IQQQ,LL)        NR4  525
0211              C(IP,IPP)=C(IPP,IP)                                                  NR4  526
0212              IF(L.EQ.LL) GO TO 113                                                NR4  527
0213              C(IPQ,IPPQ)=C(IPQ,IPPQ)- FLOAT(N(IQQQ))*D(IQ)*G(IQQ,L)*G(IQQQ,LL)   NR4  528
0214              C(IPPQ,IPQ)=C(IPQ,IPPQ)                                              NR4  529
0215          113 R(LL)=R(LL)-2                                                        NR4  530
0216              R(M)=R(M)+1                                                          NR4  531
0217              IF(R(LL).LT.0) GO TO 114                                             NR4  532
0218              IQQQ=INDEX(R,IC,M)                                                   NR4  533
0219              C(IPP,IP)=C(IP,IPP)+ FLOAT(N(IQQ))*H(IQ)*G(IQQ,L)*AK(IQQQ)*         NR4  534
                 * G(IQQ,LL)                                                          NR4  535
0220              C(IP,IPP)=C(IPP,IP)                                                  NR4  536
0221          172 IF(L.EQ.LL) GO TO 114                                                NR4  537
0222              C(IPQ,IPPQ)=C(IPQ,IPPQ)+ FLOAT(N(IQQ))*H(IQQQ)*G(IQQ,LL)*AK(IQ)*    NR4  538
                 * G(IQQ,L)                                                           NR4  539
0223              C(IPPQ,IPQ)=C(IPQ,IPPQ)                                              NR4  540
0224          114 R(LL)=R(LL)+1                                                        NR4  541
0225           69 CONTINUE                                                             NR4  542
0226              R(L)=R(L)-1                                                          NR4  543
0227           68 CONTINUE                                                             NR4  544
0228              R(M)=R(M)+1                                                          NR4  545
0229          132 CONTINUE                                                             NR4  546
0230          251 CONTINUE                                                             NR4  547
0231              IF(IC.LT.KH) GOTO 1118                                               NR4  548
0232              IF(R(1).NE.K-IC) GO TO 115                                           NR4  549
0233              GOTO 1119                                                            NR4  550
0234         1118 IF(R(N2).NE.K-IC) GOTO 1115                                          NR4  551
0235         1119 CONTINUE                                                            NR4  552
0236              PQ=BINOM(M-2,M-2+K-IC)                                               NR4  553
0237              IP=PQ                                                                NR4  554
0238              DO 70 IQ=1,IP                                                        NR4  555
0239              AKK(IQ)=AK(IQ)                                                       NR4  556
0240              DD(IQ)=D(IQ)                                                         NR4  557
0241              HH(IQ)=H(IQ)                                                         NR4  558
0242           70 CONTINUE                                                             NR4  559
0243              IF(IC-KH)1313,1314,1315                                              NR4  560
0244         1315 CONTINUE                                                            NR4  561
0245              IF(R(1).EQ.K-1) GO TO 63                                             NR4  562
0246              IC=IC-1                                                              NR4  563
0247          115 CONTINUE                                                             NR4  564
0248              CALL VEC (R,KEY,M)                                                   NR4  565
0249              GO TO 108                                                            NR4  566
0250         1314 R(1)=K-1                                                            NR4  567
0251              IF(ICD.EQ.1) GOTO 6664                                               NR4  568
0252              DO 1316 L=2,M                                                        NR4  569
0253         1316 R(L)=0                                                              NR4  570
0254              R(M)=1                                                               NR4  571
0255              DO 6660 IQ=1,N6                                                      NR4  572
0256              HH(IQ)=D0                                                            NR4  573
0257              DD(IQ)=D0                                                            NR4  574
0258              AKK(IQ)=D0                                                           NR4  575
0259         6660 CONTINUE                                                            NR4  576
0260              IC=1                                                                 NR4  577
0261              GOTO 108                                                             NR4  578
```

```
FORTRAN IV G LEVEL  18                  ESTIM                DATE = 72040          12/54/42

0262          1313 IC=IC+1                                                                  NR4  579
0263          1115 CALL AVEC(R,KEY,M)                                                       NR4  580
0264               IF(IC-KH)108,6663,6663                                                   NR4  581
0265          6663 CONTINUE                                                                 NR4  582
0266               ICD=1                                                                    NR4  583
0267               GOTO 108                                                                 NR4  584
0268          6664 CONTINUE                                                                 NR4  585
0269          6666 FORMAT(1X,5I3)                                                           NR4  586
0270          6665 FORMAT(1X,10F10.3)                                                       NR4  587
0271            63 CONTINUE                                                                 NR4  588
0272            57 CONTINUE                                                                 NR4  589
0273           201 CONTINUE                                                                 NR4  590
              C                                                                             NR4  591
              C   SECOND R-RUN IS TERMINATED.                                             NR4  592
              C                                                                             NR4  593
0274               KA=K-1                                                                   NR4  594
0275               DO 71 I=1,KA                                                             NR4  595
0276               MB=M-1                                                                   NR4  596
0277               DO 71 L=1,MB                                                             NR4  597
0278               IP=(I-1)*(M-1)+L                                                         NR4  598
0279               IQ=(K-1)*(M-1)+L                                                         NR4  599
0280               F(IP)=F(IP)*DELTA(I,L)-F(IQ)*DELTA(K,L)+ FLOAT(IA(K,L)-IA(I,L))          NR4  600
0281           203 CONTINUE                                                                 NR4  601
0282               KC=K-1                                                                   NR4  602
0283           533 DO 71 IH=1,KC                                                            NR4  603
0284               MD=M-1                                                                   NR4  604
0285               DO 71 J=1,MD                                                             NR4  605
0286               IPP=(IH-1)*(M-1)+J                                                       NR4  606
0287               IQQ=(K-1)*(M-1)+J                                                        NR4  607
0288               CC(IP,IPP)=C(IP,IPP)*DELTA(I,L)*DELTA(IH,J)-C(IP,IQQ)*DELTA(I,L)*DNR4  608
                  1ELTA(K,J)-C(IQ,IPP)*DELTA(K,L)*DELTA(IH,J)+C(IQ,IQQ)*DELTA(K,L)*DENR4  609
                  2LTA(K,J)                                                                NR4  610
0289            71 CONTINUE                                                                NR4  611
0290           902 FORMAT(1X,10F13.1)                                                      NR4  612
0291           904 CONTINUE                                                                NR4  613
0292           205 CONTINUE                                                                NR4  614
0293           900 CONTINUE                                                                NR4  615
0294          1307 FORMAT (1X,2I3,10F8.3)                                                  NR4  616
0295          1308 FORMAT(1X,8F10.4/8F10.4/8F10.4)                                         NR4  617
0296               RETURN                                                                   NR4  618
0297               END                                                                      NR4  619
```

```
FORTRAN IV G LEVEL  18                  LOGLIK          DATE = 72040        12/54/42

0001                  SUBROUTINE LOGLIK (G,IX,EPS,K,M,N8,ALR,H,N,N2)                NR5  100
                C     COMPUTES ALR= CONDITIONAL LOG-LIKELIHOOD RATIO MULTIPLIED BY  -2.NR5  101
                C     G IS MATRIX OF GAMMA-RATIOS. IX AND N CONTAINS ITEM AND PERSON NR5  102
                C     TOTALS. EPS  IS MATRIX OF ITEM PARAMETERS ON LOGARITMIC FORM. NR5  103
                C                                                                   NR5  104
                C     K,M,N2 AND N8 SPECIFIES DIMENSIONS OF MATRICES.               NR5  105
0002                  DIMENSION G(N8,N2),IX(K,M),EPS( K,M),H(N8),N(N8)              NR5  106
0003                  DIMENSION R(4)                                                NR5  107
0004                  INTEGER R                                                     NR5  108
0005                  DOUBLE PRECISION G,EPS,H,ALR,DFLOAT                           NR5  109
0006                  KEY=1                                                         NR5  110
0007                  M1=M-1                                                        NR5  111
0008                  DO 1 L=1,M1                                                   NR5  112
0009                1 R(L)=0                                                        NR5  113
0010                  R(M)=K                                                        NR5  114
0011                  ALR=0.0D0                                                     NR5  115
0012                  H(1)=0.0D0                                                    NR5  116
0013                2 IF(R(1).EQ.K)GOTO 5                                           NR5  117
0014                  CALL VEC(R,KEY,M)                                             NR5  118
0015                  IQ=INDEX(R,0,M)                                               NR5  119
0016                  R(M)=R(M)+1                                                   NR5  120
0017                  DO 3 L=1,M1                                                   NR5  121
0018                  IF(R(L).EQ.0)GOTO 3                                           NR5  122
0019                  R(L)=R(L)-1                                                   NR5  123
0020                  GOTO 4                                                        NR5  124
0021                3 CONTINUE                                                      NR5  125
0022                4 IP=INDEX(R,0,M)                                             NR5  126
0023               10 FORMAT(1X,10HGAMMAINDEX,6I4)                                NR5  127
0024                  R(L)=R(L)+1                                                   NR5  128
0025                  H(IQ)=H(IP)-DLOG(G(IQ,L))                                     NR5  129
0026                  ALR=ALR+DFLOAT(N(IQ))*H(IQ)                                   NR5  130
0027                  R(M)=R(M)-1                                                   NR5  131
0028                  GOTO 2                                                        NR5  132
0029                5 DO 6 I=1,K                                                    NR5  133
0030                  DO 6 L=1,M                                                    NR5  134
0031                6 ALR=ALR-DFLOAT(IX(I,L))*EPS(I,L)                            NR5  135
0032                  ALR=ALR+ALR                                                   NR5  136
0033                  RETURN                                                        NR5  137
0034                  END
```

```
FORTRAN IV G LEVEL 18                PSI(X)              DATE = 72040          12/54/42

0001            SUBROUTINE PSI(NC,DF,N10,KS,N,CC,F)
                C     SOLVE LIKELIHOOD EQUATIONS FROM COEFFICIENT MATRIX DC AND RIGHT HAND NR5  139
                C     SIDE F. DF RETURNS F CONTAINS SOLUTIONS. N10 IS NUMBER OF EQUATIONS. NR5  140
                C      KS=1 IF DC IS SINGULAR.                                             NR5  141
0002                  DIMENSION CC(N10,N10),DC(N10,N10),DF(N10),F(N10)                     NR5  142
0003                  DOUBLE PRECISION CC,F,DET,TLL,DF,DC,DMAX                             NR5  143
0004                  TLL=.1D-8                                                            NR5  144
0005                  DMAX=DC(1,1)                                                         NR5  145
0006                  DO 10 I=1,N10                                                        NR5  146
0007                  DO 10 J=1,N10                                                        NR5  147
0008               10 DC(I,J)=DC(I,J)/DMAX                                                 NR5  148
0009                  CALL CHOLES(N10,N10,DC,DC,DF,DET,3,TLL,IER)                          NR5  149
0010                  WRITE(6,12)                                                          NR5  150
0011               12 FORMAT(1H1,17HCOVARIANCE MATRIX)                                     NR5  151
0012               13 FORMAT(1H0,14HVARIANCE ERROR)                                        NR5  152
0013               14 FORMAT(1H0,15HSTANDARD ERRORS)                                       NR5  153
0014                  N1=N10                                                               NR5  154
0015                  DO 4 I=1,N10                                                         NR5  155
0016                  DO 11 J=1,N10                                                        NR5  156
0017               11 DC(I,J)=DC(I,J)/DMAX                                                 NR5  157
0018                  WRITE(6,5)(DC(I,J),J=1,N10)                                          NR5  158
0019                4 CONTINUE                                                             NR5  159
0020                5 FORMAT(1X,10F8.3)                                                    NR5  160
0021                  DO 6 I=1,N10                                                         NR5  161
0022                  IF(DC(I,I).LE..0D0)GOTO 16                                           NR5  162
0023                  X=DC(I,I)                                                            NR5  163
0024                  X=SQRT(X)                                                            NR5  164
0025                  DF(I)=X                                                              NR5  165
0026                  GOTO 6                                                               NR5  166
0027               15 WRITE(6,13)                                                          NR5  167
0028                6 CONTINUE                                                             NR5  168
0029                  WRITE(6,14)                                                          NR5  169
0030                  WRITE(6,5)DF                                                         NR5  170
0031                  KS=IER                                                               NR5  171
0032                  DO 1 I=1,N1                                                          NR5  172
0033                  DF(I)=0.                                                             NR5  173
0034                  DO 1 J=1,N1                                                          NR5  174
0035                1 DF(I)=DF(I)+DC(I,J)*F(J)                                             NR5  175
0036                  DO 2 I=1,N1                                                          NR5  176
0037                2 F(I)=DF(I)                                                           NR5  177
0038                  RETURN                                                               NR5  178
0039                  END                                                                  NR5  179
```

```
FORTRAN IV G LEVEL  18                  BINOV              DATE = 72040          12/54/42

0001                  FUNCTION BINOV (IX,IN)                                        NR4  181
                C     COMPUTES BINOMIAL COFFFICIENTS OF IN OVER IX. BINOM EQUAL TO  NR4  182
                C     1 FOR IX=0                                                    NR4  183
                C     0 FOR IX GREATER THAN IN EXCEPT WHEN IX=0                     NR4  184
0002                  IF(IX.EQ.0)GOTO 2                                             NR4  185
0003                  IF(IX-IN)1,2,3                                                NR4  186
0004                1 IP=1                                                          NR4  187
0005                  IR=1                                                          NR4  188
0006                  DO 4 I=1,IX                                                   NR4  189
0007                4 IP=IP*(IN-I+1)                                                NR4  190
0008                  DO 5 K=1,IX                                                   NR4  191
0009                5 IR=IR*(IX-K+1)                                                NR4  192
0010                  BINOV=FLOAT(IP/IR)                                            NR4  193
0011                  RETURN                                                        NR4  194
0012                2 IP=1                                                          NR4  195
0013                  BINOV = FLOAT(IP)                                             NR4  196
0014                  RETURN                                                        NR4  197
0015                3 IP=0                                                          NR4  198
0016                  BINOV=FLOAT(IP)                                               NR4  199
0017                  RETURN                                                        NR4  200
0018                  END                                                           NR4  201
```

```
FORTRAN IV G LEVEL  18                  INDEX              DATE = 72040          12/54/42

0001                  FUNCTION INDEX (R,IC,V)                                       NR4  202
                C     INDEX HAS DIFFERENT VALUES FOR IC=0 AND OTHERWISE             NR4  203
                C     IC=0 INDEX= INDEX OF INTEGER V=VECTOR R IN COMPLETE LISTING OF NR4  204
                C     POSSIBLE VALUES                                               NR4  205
                C     IC POSITIVE INDEX= INDEX OF R IN SUBLISTING OF ALL R WITH R(V)=IC.NR4  206
0002                  INTEGER S(4),R(10)                                            NR4  207
0003                  COMMON V0,K,NA                                                NR4  208
0004                  IF(V.EQ.2)GOTO 97                                             NR4  209
0005                  IF(IC-0)99,99,3                                               NR4  210
0006               99 S(1)=R(1)                                                     NR4  211
0007                  V1=V-1                                                        NR4  212
0008                  DO 1 L=2,V1                                                   NR4  213
0009                  V10=V-L                                                       NR4  214
0010                1 S(L)=S(L-1)+R(V10+1)                                          NR4  215
0011                  PL=1.                                                         NR4  216
0012                  DO 2 L=1,V1                                                   NR4  217
0013                2 PL=PL+BINOV(V-L,V-1-L+K-S(L))                                 NR4  218
0014                  INDEX=IFIX(PL+0.5)                                            NR4  219
0015                  RETURN                                                        NR4  220
0016                3 S(1)=R(V-1)                                                   NR4  221
0017                  V2=V-2                                                        NR4  222
0018                  IF(V2-2)6,98,98                                               NR4  223
0019               98 DO 4 L=2,V2                                                   NR4  224
0020                  V10=V-L                                                       NR4  225
0021                4 S(L)=S(L-1)+R(V10)                                            NR4  226
0022                6 PL=1.                                                         NR4  227
0023                  DO 5 L=1,V2                                                   NR4  228
0024                5 PL=PL+BINOV(V-L-1,V-L-2+K-IC-S(L))                            NR4  229
0025                  INDEX=IFIX(PL+0.5)                                            NR4  230
0026                  RETURN                                                        NR4  231
0027               97 IF(IC)95,95,96                                                NR4  232
0028               96 INDEX=1                                                       NR4  233
0029                  RETURN                                                        NR4  234
0030               95 INDEX=R(1)+1                                                  NR4  235
0031                  RETURN                                                        NR4  236
0032                  END                                                           NR4  237
```

```
FORTRAN IV G LEVEL  18                  VEC                  DATE = 72040          12/54/42

0001                  SUBROUTINE VEC(R,KEY,N)                                          NR4  238
                C     CHANGE INTEGER N-VECTOR R INTO NEXT R IN COMPLETE LISTING.       NR4  239
                C     KEY=1 OR2 EXCEPT WHEN R(N)=0. THEN =3.                           NR4  240
0002                  INTEGER R(10)                                                    NR4  241
0003                  N1=N-1                                                           NR4  242
0004                  J=R(1)                                                           NR4  243
0005                  R(1)=0                                                           NR4  244
0006                  IF(N.EQ.2)GOTO 5                                                 NR4  245
0007                  DO 2 I=2,N1                                                      NR4  246
0008                  IF(R(I))2,2,1                                                    NR4  247
0009                1 R(I-1)=1+J                                                       NR4  248
0010                  R(I)=R(I)-1                                                      NR4  249
0011                  RETURN                                                           NR4  250
0012                2 CONTINUE                                                         NR4  251
0013                5 CONTINUE                                                         NR4  252
0014                  IF(R(N))3,3,4                                                    NR4  253
0015                3 KEY=3                                                          NR4  254
0016                  RETURN                                                           NR4  255
0017                4 R(N-1)=1+J                                                       NR4  256
0018                  R(N)=R(N)-1                                                      NR4  257
0019                  KEY=3-KEY                                                        NR4  258
0020                  RETURN                                                           NR4  259
0021                  END                                                              NR4  260
```

```
FORTRAN IV G LEVEL  18                  AVEC                 DATE = 72040          12/54/42

0001                  SUBROUTINE AVEC(R,KEY,N)                                         NR4  261
                C     REVERS ROUTINE OF VEC. CF. COMMENTS FOR THIS.                    NR4  262
0002                  DIMENSION R(N),IS(4)                                             NR4  263
0003                  INTEGER R                                                        NR4  264
0004                  COMMON MG,K,NA                                                   NR4  265
0005                  N1=N-1                                                           NR4  266
0006                  IS(N)=R(N)                                                       NR4  267
0007                  DO 9 LL=1,N1                                                     NR4  268
0008                  L=N-LL                                                           NR4  269
0009                9 IS(L)= R(L)+IS(L+1)                                              NR4  270
0010                  IF(R(1))10,10,11                                                 NR4  271
0011               11 R(1)=R(1)-1                                                      NR4  272
0012                  R(2)=R(2)+1                                                      NR4  273
0013                  GOTO 13                                                          NR4  274
0014               10 DO 12 L=2,N                                                      NR4  275
0015                  IF(K-IS(L)) 12,12,18                                             NR4  276
0016               18 R(L)=R(L)+1                                                      NR4  277
0017                  R(1)=K-IS(L)-1                                                   NR4  278
0018                  L1=L-1                                                           NR4  279
0019                  IF(L1-2)13,15,15                                                 NR4  280
0020               15 DO 14 LL=2,L1                                                    NR4  281
0021               14 R(LL)=0                                                          NR4  282
0022                  GOTO 13                                                          NR4  283
0023               12 CONTINUE                                                         NR4  284
0024               13 IF(K-R(1))2,2,1                                                  NR4  285
0025                2 KEY=3                                                          NR4  286
0026                  RETURN                                                           NR4  287
0027                1 KEY=3-KEY                                                        NR4  288
0028                  RETURN                                                           NR4  289
0029                  END                                                              NR4  290
```

```
FORTRAN IV G LEVEL 18                    CHOLES                DATE = 72040          12/54/42

0001                SUBROUTINE CHOLES(IA,N,A,B,Y,D,IND,EPS,IERR)                          NR5  201
                C   FOR A GIVEN SYMMETRIC, POSITIVE DEFINITE MATRIX A OF ORDER N         NR5  202
                C   THIS SUBROUTINE COMPUTES B=INVERSE OF A. ONLY THE LOWER HALF        NR5  203
                C   OF A IS USED. THE METHOD OF INVERSION IS CHOLESKYS SQUARE ROOT      NR5  204
                C   METHOD AS DESCRIBED BY DICKMAN AND KAISER, EDUC. PSYCH. MEASMT.,    NR5  205
                C   21,1961,721-727. IF ANY PIVOTAL ELEMENT IS LESS THAN EPS, THE      NR5  206
                C   MATRIX A IS CONSIDERED SINGULAR AND CONTROL IS TRANSFERRED TO THE   NR5  207
                C   CALLING PROGRAM WITH IERR=1. OTHERWISE IERR IS ZERO.               NR5  208
                C   IF IND=1, THE DETERMINANT OF A IS STORED IN D AND THE NEGATIVE      NR5  209
                C   OF THE INVERSE IS STORED IN THE LOWER HALF OF B.                   NR5  210
                C   IF IND=2, THE INVERSE IS STORED IN THE LOWER HALF OF B.            NR5  211
                C   IF IND=3, THE FULL INVERSE IS STORED IN B.                        NR5  212
                C   IF THE MATRIX A CAN BE DESTROYED, A AND B CAN BE THE SAME MATRIX.  NR5  213
0002                DOUBLE PRECISION A,B,Y,D,F,EPS,H                                    NR5  214
0003                DIMENSION A(IA,1),B(IA,1),Y(IA)                                     NR5  215
0004                IERR=0                                                              NR5  216
0005                DO 5 I=1,N                                                          NR5  217
0006                DO 5 J=1,I                                                          NR5  218
0007              5 B(I,J)=A(I,J)                                                       NR5  219
0008                D=1.                                                                NR5  220
0009                DO 40 L=1,N                                                         NR5  221
0010                F=B(L,1)                                                            NR5  222
0011                IF(DABS(F).LT.EPS) GOTO 120                                         NR5  223
0012                D=D*F                                                               NR5  224
0013                F=1./F                                                              NR5  225
0014                DO 20 I=1,N                                                         NR5  226
0015             20 Y(I)=B(I,1)                                                         NR5  227
0016                DO 30 I=2,N                                                         NR5  228
0017                H=Y(I)*F                                                            NR5  229
0018                DO 30 J=2,I                                                         NR5  230
0019             30 B(I-1,J-1)=B(I,J)-Y(J)*H                                            NR5  231
0020                DO 35 J=2,N                                                         NR5  232
0021             35 B(N,J-1)=-Y(J)*F                                                    NR5  233
0022             40 B(N,N)=-F                                                           NR5  234
0023                IF(IND-1) 110,110,50                                                NR5  235
0024             50 DO 60 I=1,N                                                         NR5  236
0025                DO 60 J=1,I                                                         NR5  237
0026             60 B(I,J)=-B(I,J)                                                      NR5  238
0027                IF(IND-2) 110,110,55                                                NR5  239
0028             55 DO 70 I=2,N                                                         NR5  240
0029                I1=I-1                                                              NR5  241
0030                DO 70 J=1,I1                                                        NR5  242
0031             70 B(J,I)=B(I,J)                                                       NR5  243
0032            110 RETURN                                                              NR5  244
0033            120 WRITE (6,1) L,B(1,1),D                                              NR5  245
0034                IERR=1                                                              NR5  246
0035                RETURN                                                              NR5  247
0036              1 FORMAT (37HOTHE MATRIX IS NOT POSITIVE DEFINITE ,I4,2F14.8)         NR5  248
0037                END                                                                 NR5  249
```

```
FORTRAN IV G LEVEL  18                  PROB                DATE = 72040        12/54/42

0001                  SUBROUTINE PROB(IX,P,Y,JJ)                                    NR5  180
                C     GENERATE RANDOM MULTINOMIAL VARIABLE BETWEEN 1 AND Y FOR GIVE   NR5  181
                C     PROBABILITY VECTOR P.                                         NR5  182
0002                  DIMENSION P(Y)                                                NR5  183
0003                  CALL RANDU(J,JJ,R)                                            NR5  184
0004                  J=JJ                                                          NR5  185
0005                  Q=0.                                                          NR5  186
0006                  DO 1 I=1,Y                                                    NR5  187
0007                  Q=Q+P(I)                                                      NR5  188
0008                  IF (Q-R)1,2,2                                                 NR5  189
0009                2 IX=I                                                          NR5  190
0010                  RETURN                                                        NR5  191
0011                1 CONTINUE                                                      NR5  192
0012                  RETURN                                                        NR5  193
0013                  END                                                           NR5  194


FORTRAN IV G LEVEL  18                  DFLOAT              DATE = 72040        12/54/42

0001                  FUNCTION DFLOAT(I)                                            NR5  195
                C     DOUBLE PRECISION VERSION OF FLOAT                             NR5  196
                                                                                    NR5  197
0002                  DOUBLE PRECISION DFLOAT                                       NR5  198
0003                  DFLOAT=I                                                      NR5  199
0004                  RETURN                                                        NR5  200
0005                  END
```